APPM 3310: Matrix Methods — Exam #1 — February 20, 2009

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. **Explain all of your answers.** A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. No books or notes. No electronic devices of any kind (e.g. cell phones, calculators, etc.) are permitted. Begin each problem on a new page.

1. (40 points) For this problem, let $A = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 6 & 2 & 5 & 14 \\ -2 & -3 & 10 & 0 \end{bmatrix}$

   (a) Find the LU decomposition of A where U is in row echelon form.

   (b) Find $\ker(A)$ and $\text{rng}(A)$ by exhibiting a basis for each.

   (c) Determine the rank of A and the dimensions of the four fundamental subspaces associated with A.

   (d) Let $\mathbf{b} = [0 \quad 3 \quad 6]^T$. Use the LU decomposition from the previous part to find a solution to the linear system $A\mathbf{x} = \mathbf{b}$

2. (40 points) For $\mathbb{M}_{n\times n}$, the set of all $n \times n$ matrices with entries in $\mathbb{R}$ answer the following.

   (a) Show that $\mathbb{M}_{n\times n}$ is a vector space.

   (b) Exhibit a basis for and state the dimension of $\mathbb{M}_{n\times n}$

   (c) Recall that for an $n \times n$ matrix A, $\text{tr}A = \sum_{i=1}^{n} a_{ii}$. Let $A, B \in \mathbb{M}_{n\times n}$.

      i. *True or False*: $\text{tr}(A + B) = \text{tr}(B + A)$. Prove or give a counterexample.

      ii. Let $T = \{A \in \mathbb{M}_{n\times n} | \ \text{tr}A = 1\}$. Is $T \subset \mathbb{M}_{n\times n}$ a **subspace?**

3. (30 points) Answer the following True/False questions by either showing why the statement is true in general or providing a counter example to show that it is false.

   (a) *True or False* An $n \times n$ matrix A has rank n if and only if $kerA = \{\mathbf{0}\}$.

   (b) *True or False* If the zero vector belongs to the span of a collection of nonzero vectors then the vectors are linearly dependent.

   (c) *True or False* If A is an $m \times n$ matrix and $rngA = \mathbb{R}^m$ then $kerA = \mathbf{0}$.