APPM 3310: Matrix Methods — Exam #2 — April 8, 2009

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. **Explain all of your answers.** A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. No books or notes. No electronic devices of any kind (e.g. cell phones, calculators, etc.) are permitted. Begin each problem on a new page.

1. (20 points) Let

$$A = \begin{bmatrix} 1 & -1 & 2 & 5 \\ 1 & 2 & 1 & -1 \\ 1 & -1 & 0 & -7 \\ 1 & 0 & 3 & 3 \end{bmatrix}$$

   Given that A is nonsingular, find the QR factorization of A.

2. (20 points) Consider $\mathbb{R}^3$ with the standard Euclidean dot product. Let $V \in \mathbb{R}^3$ be the two-dimentional subspace spanned by $\mathbf{v}_1 = (1. -1, 1)^T$, $\mathbf{v}_2 = (-1, 1, 2)^T$ and let $\mathbf{b} = (1, 0, 2)$.

   (a) Find $\mathbf{v}^* \in V$ that minimizes the distance from $\mathbf{b}$ to V using any method discussed in lecture.

   (b) Find d^* the distance from $\mathbf{b}$ to V.

3. (50 points) Let K be a symmetric matrix.

   (a) Suppose K is nonsingular, prove that if K is positive definite then so is K^{-1}.

   (b) *True or False* If K admits a null direction, then $ker(K) \neq \mathbf{0}$.

   (c) Find the Gram matrix K for the monomials 1, x, x^2 under the L^2 inner product on [0,1].

   (d) Prove that if K is positive definite it can be written as a Gram matrix.

   (e) Determine whether the quadratic form $q(\mathbf{x}) = 2x_1^2 + x_1x_2 - 2x_1x_3 + 2x_2^2 - 2x_2x_3 + 2x_3^2$ is positive definite on $\mathbb{R}^3$ by writing it as a sum of squares. (Hint: Use matrices and complete the square!)

4. (20 points)

   (a) Let $V = C^1[-1, 1]$ denote the vector space of continuously differentiable functions on [0,1]. Does

$$< f, g >= \int_{-1}^{1} f'(x)g'(x)dx \tag{1}$$

   define an inner product on V? Show that all three inner product axioms hold or show at least one that does not hold.

   (b) Let $\|\mathbf{v}\|_1$ and $\|\mathbf{v}\|_2$ be two different norms on a vector space V. Does the arithmetic mean

$$\|\mathbf{v}\| = \frac{1}{2}(\|\mathbf{v}\|_1 + \|\mathbf{v}\|_2) \tag{2}$$

   define a norm? Show that all three norm axioms hold or show at least one that does not hold.