APPM 4360/5360 - Review for Exam 2

1. Analytic functions and applications

   - Given the real or imaginary part of an analytic function, be able to determine its harmonic conjugate and the original function using the Cauchy-Riemann equations.
   - Be able to verify that the real and imaginary parts of an analytic function satisfy Laplace's equation.
   - Given a complex velocity potential of the form

     $$\Omega(x+iy) = \phi(x,y) + i\psi(x,y),$$

     be able to identify the stream function, potential function, and draw the level curves of these functions for simple cases. Be able to interpret these level curves in terms of ideal fluid flow.

2. Integration

   - Be able to compute integrals of functions along simple paths in the complex plane (i.e. curves composed of arcs and straight lines) via parameterization.
   - Be able to state the fundamental theorem of calculus for functions of a complex variable and be able to apply it.
   - Be able to state the Cauchy-Goursat theorem.
   - Be able to state Morera's theorem.
   - Understand the deformation principle

     If a curve can be deformed without passing through a function's singularities, then the value of the function's integral along that curve does not change with the deformation.

     Be able to apply this when convenient.

- Be able to state Cauchy's integral formula and its derivative form.

$$f^{(k)}(z) = \frac{k!}{2\pi i} \int_C \frac{f(\xi)}{(\xi - z)^{k+1}} d\xi$$

  where z is an interior point to the simple closed curve C and ξ is a point on the curve.
- Be able to state Liouville's theorem.
- Be able to state and apply the maximum/minimum modulus principle for analytic functions.
- Be able to state and apply the maximum/minimum principle for real valued harmonic functions.

3. Power series

- Be able to state Taylor's theorem for analytic functions, including the formula for the coefficients.
- Be able to use the comparison test, ratio test, and root test to determine the radius of convergence of a Taylor series.
- Know what it means for the Taylor series to converge uniformly. Know what implications this has for differentiating and integrating term by term.
- Know the Taylor series for elementary functions such as e^z, $sin(z)$, $cos(z)$, $ln(1+z)$, and $\frac{1}{1-z}$.
- Be able to state Laurent's theorem.
- Be able to determine the Laurent series expansion for a function about its singularities.
- Using Laurent series, be able to classify a singularity as a pole, essential singularity, or removable singularity.
- Be able to give the Laurent series expansion of a function in a given region of the complex plane.
- Be able to classify singularities at infinity.