Summary of the number of solutions for a linear system $A_{m\times n}\, x_n = b_m$ where $m < n,\ m = n,$ or $m > n.$

From splitting $A_{m\times n}$ as $A = U\,\Sigma\, V^*$ follows that it suffices to analyze the issue for linear systems $\Sigma_{m\times n}\, x_n = b_m$ where Σ is a singular value matrix.

	Full rank	**Partial rank**
Underdetermined $\begin{bmatrix}\ddots & & & & \\ & \ddots & & & \end{bmatrix}\begin{bmatrix}\vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\end{bmatrix}=\begin{bmatrix}\vdots\\ \vdots\end{bmatrix}$	First m entries uniquely determined; rest undetermined ∞ many solutions	Additionally, conflict can arise if last entry (entries) of b are non-zero 0 or ∞ many solutions
Square $\begin{bmatrix}\ddots & \\ & \ddots\end{bmatrix}\begin{bmatrix}\vdots\\ \vdots\end{bmatrix}=\begin{bmatrix}\vdots\\ \vdots\end{bmatrix}$	1 solution	Same as above 0 or ∞ many solutions
Overdetermined $\begin{bmatrix}\ddots & \\ & \ddots\\ & \\ & \end{bmatrix}\begin{bmatrix}\vdots\\ \vdots\end{bmatrix}=\begin{bmatrix}\vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\end{bmatrix}$	All unknowns uniquely determined. But conflict if any of the last *m-n* elements of b are non-zero 0 or 1 solution	Same as above 0 or ∞ many solutions