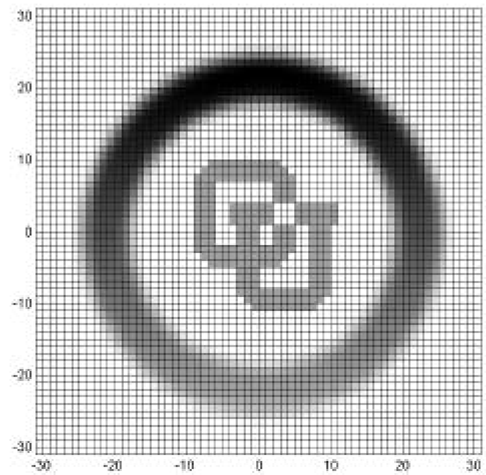

Figure 1.1: Display of test object.

1.3 Model Problem

Figures 1.1 and 1.2 show a test object, defined on a 63×63 grid. This object was generated by the code logo.m given in Section ..... The X-ray absorption levels at different locations are displayed as darkness and elevation respectively. Figure 1.3 shows how X-ray data can be collected for a sequence of angles $\theta_i = \frac{\pi\, i}{64}, i = 0, 1, ..., 63$. Figure 1.4 (computed by the code scan.m in Section ....) shows what the scan data would look like in the case of the test object in Figres 1.1 and 1.2. The scan lines are shown as successive lines (in the r-direction) from the front left ($\theta = 0$) to the back right ($\theta = \pi$; this last line being an up-down reflection of the first one).

Given the density function $f(x, y)$ of the 2-D object, the scan data can be written

$$g(r, \theta) = \int_{-\infty}^{\infty} f(x, y)\; ds \tag{1.1}$$

where the coordinate axes are as defined in Figure 1.5, satisfying

$$\left\{ \begin{array}{l} s = x\cos\theta + y\sin\theta \\ r = -x\sin\theta + y\cos\theta \end{array} \right. \qquad \left\{ \begin{array}{l} x = s\cos\theta - r\sin\theta \\ y = s\sin\theta + r\cos\theta \end{array} \right. \quad ..$$

This equation (1.1) is known as the *Radon transform.* The computational issue in CT is to invert this transform, i.e. to recover $f(x, y)$ from $g(r, \theta)$.

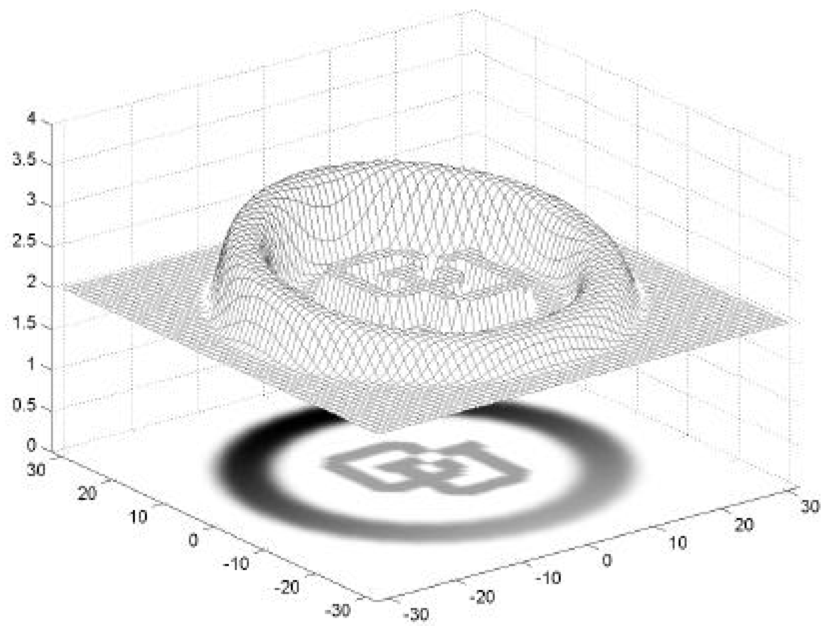

Figure 1.2: Another display of test object.

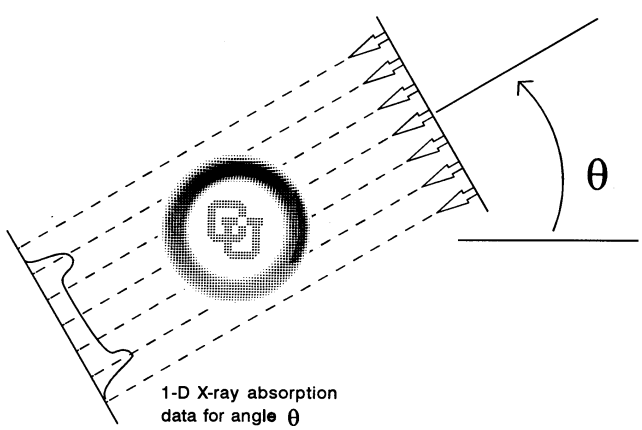

Figure 1.3: Principle for generation of 1-D scan data from a 2-D object.

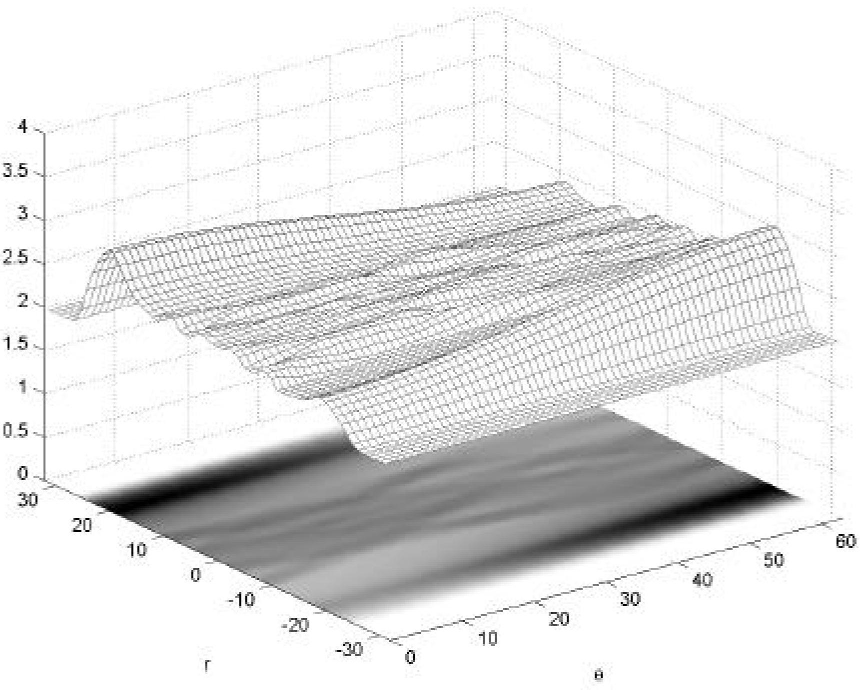

Figure 1.4: Scan data of the CU-object.

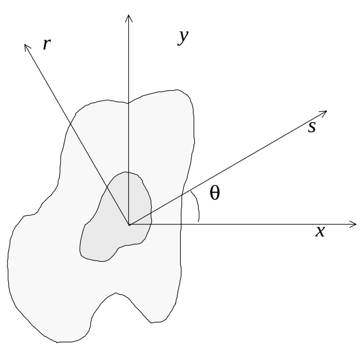

Figure 1.5: Relation between the (x, y) and (s, r) coordinate systems.