

2.4 Test Problem - Circular Current

Figure 1 shows a test problem for ray tracing with equation (3.3-11) that has been used by some investigators (Gerber, 1993, White and Fornberg, 1997) - a circular current $U(x,y)$ with inner radius 40 km and outer radius 160 km. Between these, the velocity profile (a function of radius r only) is parabolic, with a maximal velocity of 2 m/s.

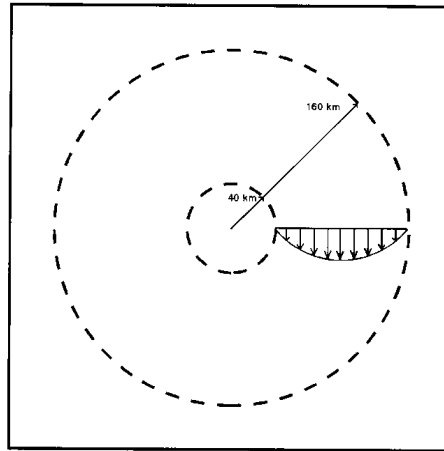


Figure 1. Test example of circular current. Parabolic velocity profile with maximum velocity of 2 m/s.

We now denote the spatial directions x and y (rather than x_1 and x_2), and the velocity components in these directions $u(x,y)$ and $v(x,y)$ respectively. Figure 2 shows the u -field (as to be expected most positive at $(0,100)$ (measured in km), most negative at $(0,-100)$ and zero along $y = 0$). In the wave ray field shown in Figure 3, each wave has a time period $T = 10$ s, i.e. according to (I2.3-5) $\sigma = 5/\pi$. From (I2.3-7) follows then $|\mathbf{k}| = 25/(\pi^2 \cdot 9.8)$. At equi-spaced positions \mathbf{x} along the left and bottom edges of the domain, we start all the rays with $\mathbf{k} = (1, 1) \cdot |\mathbf{k}|/\sqrt{2}$ (a vector of the right length, pointing NE). The computation of the paths involves the following steps:

- Generate the u - and v -fields for the circular current, and
- For each ray, solve the four coupled ODEs (I2.3-11) for as long time as needed until the ray exits the domain.

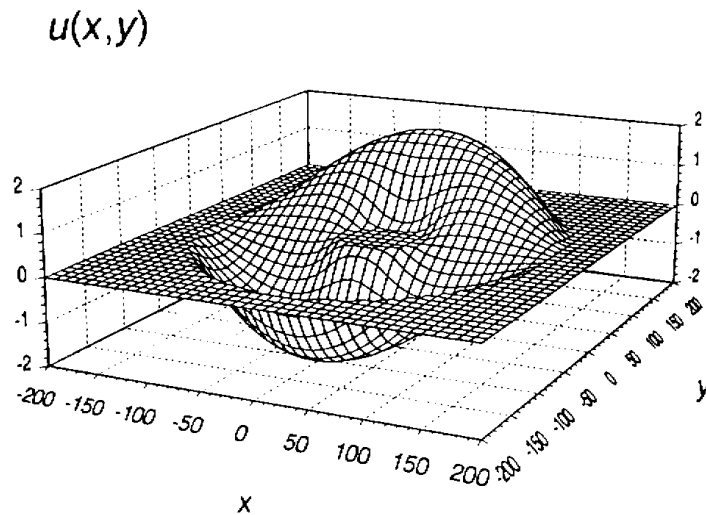


Figure 2. Velocity component in x -direction of circular current.

In the code given in Section ... , a second order, 2-stage Runge-Kutta method was used as ODE solver - many other choices would have worked equally well (cf. discussion of ODE solvers in Chapter III3). The other main numerical issue is interpolation / finite difference approximation. At every (discrete) (x,y) -position that we reach along a ray path, the values for u , v , u_x , u_y , v_x , v_y must all be obtained by interpolation from adjacent (x,y) -grid positions. The methods for this are discussed in Chapter III3.

A ray path picture like Figure 3 is very effective in conveying focus spots of wave energy. In other cases, it might be more useful to display the wave energy (roughly equal the ray density) as a function of spatial position as in Figure 4 - generated from the rays in Figure 3.

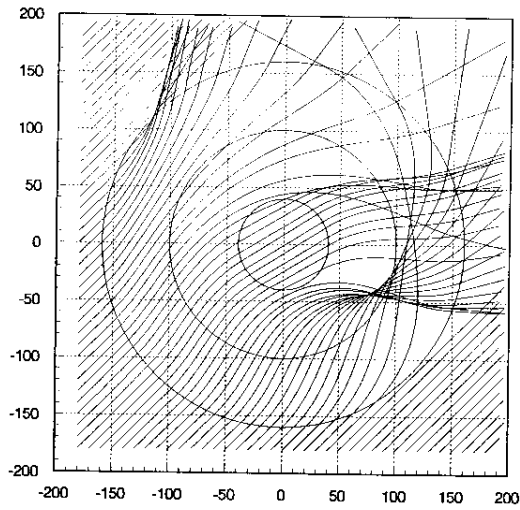


Figure 3. Ray pattern for circular current in case of incoming monochromatic waves from SW with a time period $T = 10$ s (corresponding to a wavelength $\lambda \approx 160$ m.)

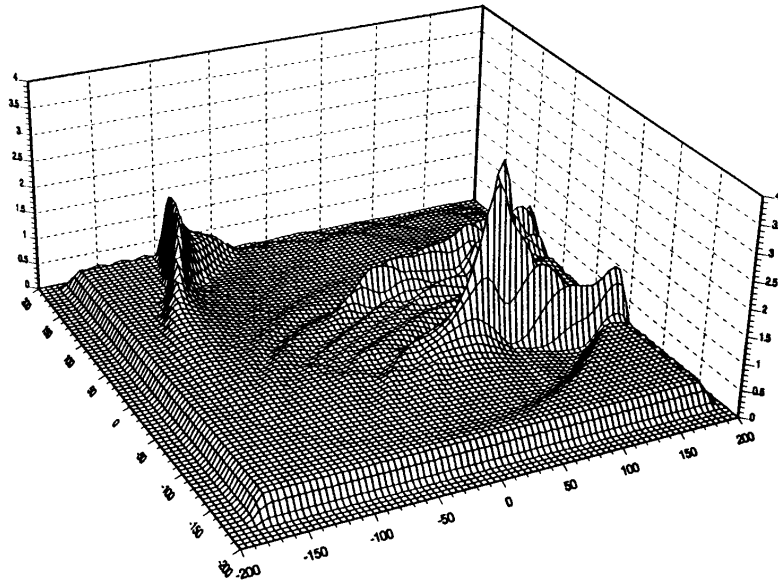


Figure 4. Ray pattern of Figure 3 translated into a surface display of ray density - an approximate measure of wave energy.