4.2 Principles of GPS

Figure 1 summarizes some technical specifications for GPS and GLONASS. The systems are very similar in most respects. Before concentrating on GPS, let's note the two principal differences:

- All the GPS satellites broadcast on exactly the same frequency in order to save bandwidth; the transmission of separate pseudorandom (PR) sequences - described below - allows this without causing signal confusion. Two GLONASS satellites exactly opposite each other use the same frequency - the 24 satellites require therefore 12 separate frequencies (every GLONASS satellite uses the same PR sequence),

- GLONASS does not degrade the signal accuracy for the public the way *selective availability* (SA) does for GPS. A small GLONASS receiver - if easily available - would probably be somewhat more accurate than a GPS one.

The first mathematical problem that arises is how to send a very sharp pulse (so that its arrival can be very accurately timed) without needing to use a wide bandwidth. The function - Fourier transform pair $f(x) = e^{-\alpha x^2},\ \hat{f}(\omega) = \frac{1}{\sqrt{4\pi\alpha}} e^{-\omega^2/(4\alpha)}$ (cf. Table 2 in Section II3.2) illustrates the dilemma. The parameter α large means that the function $f(x)$ is a sharp pulse - allowing its position to be accurately pinpointed. However, $\hat{f}(\omega)$ will then have a very broad maximum - occupying a lot of frequency space (bandwidth). The parameter α small will make the pulse broad - making it difficult to accurately determine its position (center).

The answer to this apparent conflict turns out to be that one after all does not need a sharp pulse in order to achieve a fine time resolution - a suitably structured signal of long duration can also achieve a fine time resolution. Each satellite sends its own PR signal as illustrated in Figure 2; small up/down-variations in frequency according to a pattern that looks seemingly random, but is fixed for each satellite, and repeats after 1023 *chip times* of 1 μs each. Thus, the whole pattern repeats roughly each ms. During each 1 μs chip time, the carrier (at 1575.42 MHz) goes through about 1,500 cycles - the modulation is of a relatively low frequency relative to the carrier (i.e. quite narrow bandwidth). A receiver knows the pseudo-random sequence (for each satellite), and slides its copy relative to the received signal until the match gets perfect (cf. again Figure 2). Even a small misalignment - about 1/20th of a chip time - will notably bring down the correlation. This size timing error would correspond to a distance error of about 15 m, typical of the hardware capabilities of low cost GPS receivers.

Figure 3 shows the correlation function (integral of the product of the two sequences), displayed as a function of the sideways shift in a case of a periodic PR sequence of length 128. Whenever the shift is a multiple of 128, we get here a very sharp spike of perfectly triangular shape with a base width of 2 chip times.

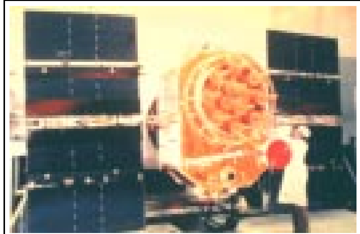
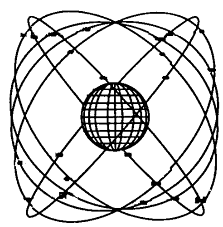
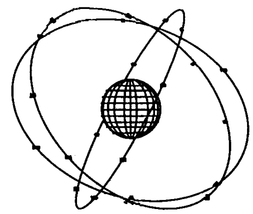
	GPS	GLONASS
		
Operated by	US DOD (Department of Defense)	Russia
Control center	Falcon Air Force Base (near Colorado Springs, CO)	?
First satellite launched	1978	1982
System operational	1993	1993
Constellation (as viewed from equatorial plane)		
Number of satellites	24 + 3 spares	24 + 2 or 3 spares
Satellite distribution	4 spaced 90^0 apart in each of six planes	8 spaced 45^0 apart in each of three planes
Orbital inclination to equatorial plane	55^0 (limited by possible orbits of the space shuttle - for launching and servicing)	64.8^0
Average elevation (from *center* of earth)	27,560 km (about 3.0 earth radii above its surface - at outer edge of upper van Allen radiation belt)	25,510 km
Orbital period	11 h 58 m (one half sidereal day)	11 h 15 m 45 s
Frequency of orbital information update from ground	every hour	every half hour
SA (Selective Availability) - a scheme to selectively degrade accuracy for civilian users.	Yes - degrades precision to around 100 m. SA may be phased out; if so giving about 20 m accuracy	No - accuracy around 20 meters
Both systems in addition carry a military-only system of about 10 times higher accuracy, operating at frequencies 10 times higher than those described here etc.	Military accuracy: < 10 m	Military accuracy: < 10 m
Radio frequencies (civilian)	1575.42 MHz Navigational information. 1227.60 MHz Control signal; allows specially equipped receivers to estimate ionospheric delays	$1602.0 + n \cdot 0.5625$ MHz $1246.0 + n \cdot 0.4377$ MHz $n = 0, 1, ..., 12$
Length of pseudorandom code	1023 (= 2^{10}-1) bits	511 (= 2^9-1) bits
Chip rate; repeat time of pseudorandom code	1.023 MHz; 1.0 ms	0.511 MHz; 1.0 ms
Data package (orbital information): rate; length	50 bits/s ; 30 s	50 bits/s ; 30 s

Figure 1. Some technical specifications of GPS and GLONASS.

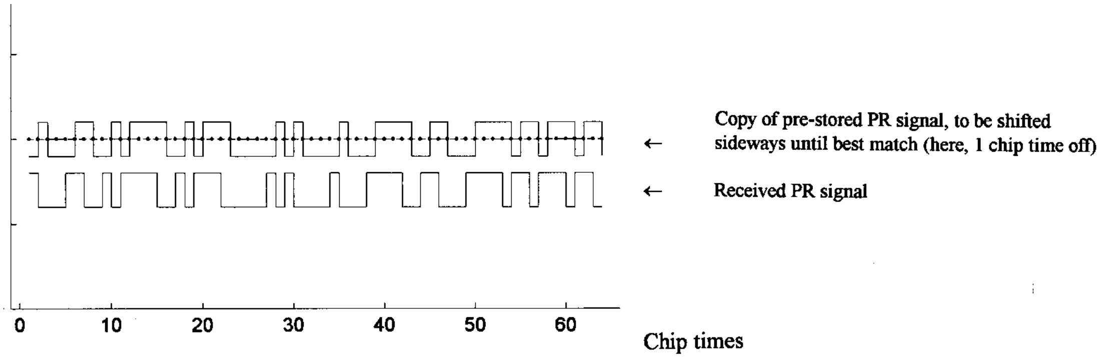


Figure 2. Pseudorandom (PR) received code compared with a copy stored in the receiver.

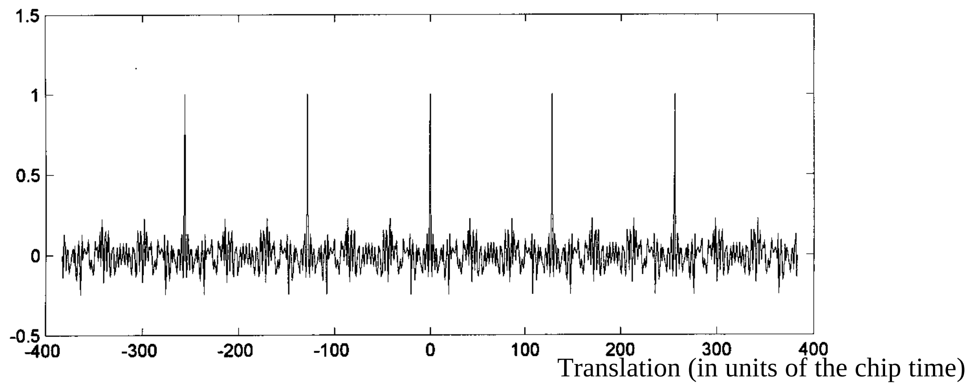


Figure 3. Correlation between a periodic PR code of length 128 and a translated copy of itself. The sharp spikes appear here every 128 chip times.

For GPS, the periodic repeat time was about 1 ms, which given the speed of light, corresponds to about 300 km. All distances to satellites become arbitrary with respect to multiples of this distance. The easiest solution to this is to require that the receiver has to be *initialized* - the user has first to enter an approximate position that is accurate to about 150 km, thus removing the arbitrariness. There are (at least) two other ways to resolve this:

i. Guessing wrong by 300 km (when receiving signals from the minimal number of satellites required to get a position) is very likely to produce a position hundreds of kilometers under ground or above ground - easily rejected for all receivers not used in space crafts, and

ii. When receiving from more satellites, a wrong guess will quite certainly produce contradictory distance readings - no single point will satisfy all the distance requirements if any reading is wrong by a multiple of 300 km.

This second observation can be turned into a numerical technique - *integer programming*. If every received distance is known apart from with respect to a multiple of 300 km, the problem becomes how to choose these integer multiples so that a single-point solution becomes possible.

As it turns out, integer programming is of great importance for GPS, but in a slightly different way. To get the highest possible accuracy, we want to utilize the individual oscillations on the 1575.42 MHz carrier - its wave length is about 0.2 m. Trying to lock on to the phase angle of the carrier oscillations, we get distances to within an unknown multiple of 0.2 m. With a good positional guess, say from *differentially corrected GPS* (described later), we may have only about 50 - 100 multiples to be concerned with. Finding a position that gives a correct carrier phase for all available satellites can pinpoint just which carrier multiple we are locked on to for each of the satellites in view, i.e. an error better than 0.2 m. Finally, being locked onto exactly the right carrier oscillation, the phase angle can be reconciled to maybe one part in 200. The accuracy is thus down to about 1 mm - not bad considering that the radio signals are of quite narrow bandwidth! In the rest of this GPS discussion, we will not pursue this issue of locking onto carrier phase angle, or integer programming - although mathematically and numerically very interesting. For material on this, see Strang and Borre (1997).

Some of the error sources for the positional calculations are described in Section 4.4. All but one of them are more or less unavoidable - the GPS system is designed to keep these as low as possible. The big exception to this is *selective availability* (SA). In order to give a battlefield advantage over potential adversaries, the military intentionally degrades the signal accuracy. The readings at any fixed place will randomly drift about 70 m in different directions; the error is less that 100 m about 95 % of the time. Special military receivers are programmed to compensate for these (highly secret) error patterns. Figure 4 shows an example of how the readings at a fixed location drift over one hour when using a military and a civilian receiver.

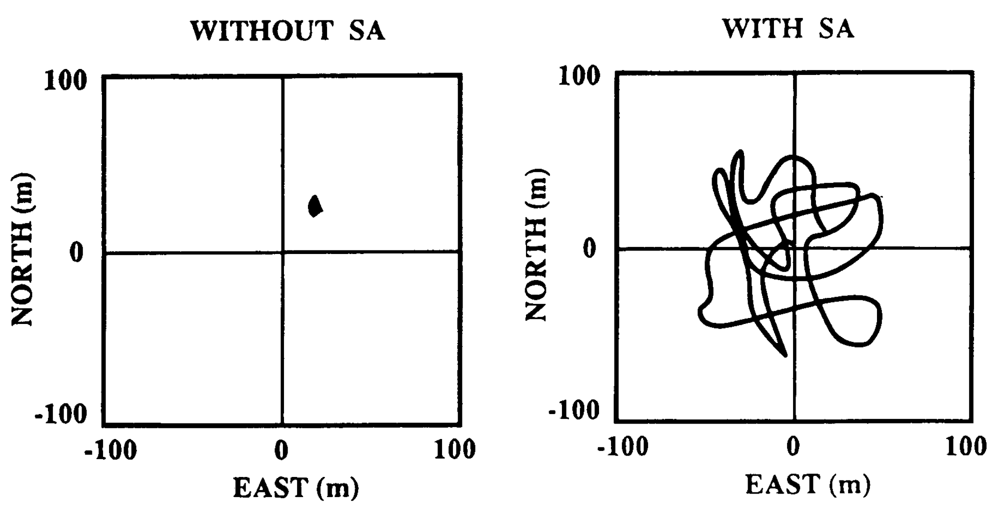


Figure 4. Comparison of errors during one hour for a military and a civilian GPS receiver.

We note that other error sources than SA contribute to a 10-20 meter error also for the military receiver. A stationary civilian receiver will typically give velocity readings of around 1-2 km/h in random directions. Since this artificial velocity is well less than even the speed of a hiker, it has little effect on measuring most velocities of interest.

> SA was turned off during the Gulf War in 1992 - one might have thought that security then would have suggested the opposite. However, there arose a big shortage of military receivers, and a large number of civilian ones were used. It was assumed that the enemies' access to that market then was too small to be significant.

President Clinton has proposed that SA be phased out - this is at present opposed by the military, and no final decision appears to have been made. In the meantime, several ways to 'defeat' SA have been developed, notably *differential correction* (DC) which gives civilian receivers significantly better accuracy than even SA-corrected ones. The idea is to place a GPS receiver at a known location, record its instantaneous readings, and broadcast (on radio) positional correction information. This also corrects for other errors than SA - such as ionospheric conditions which may affect the speed of signals. There is now nearly a full national coverage available of DC stations. However, this service has usually to be subscribed to, at some cost.