4.3 Formulation of a Test Problem

If we first assume that the receiver clocks are perfectly accurate (as we safely can assume that the transmitter clocks are), timing differences between known transmission times and detected receiving times translate directly into distance measurements.

Constructing a sphere around each satellite with the distance to it as radius, we would be located somewhere on its surface. If we receive signals from two satellites in known positions, we are somewhere on the intersection of two spheres, i.e. somewhere along a circle in space. If we are at a known height, e.g. at sea, this data would suffice to pin our position. The circle would intersect the sea surface at two points, likely very far apart, and thus making the choice easy.

If we need to determine our height as well, we need a third satellite; the sphere from it would intersect the circle from the first two at two points. One of these two points is likely way out in space, and can be disregarded. The second intersection pinpoints out position; N-S, E-W and height.

Our very first assumption is unlikely to hold - already a 1 ms error in the receiver clock would throw all distances off by 300 km. To make receivers inexpensive, thier clocks are no more accurate than a typical watch, and may well be, say, 10 s off (which would give distance errors of many times the distance to the moon - light from there takes just over one second to reach us). The way around this is to pick up singals from still one more satellite (i.e. 4 in all). If the receiver clock was perfect, the distance spheres from all 4 satellites would intersect precisely in one point; with a clock error, there qill not be not be such an intersection point for the 4 spheres. So the task becomes to adjust the receiver clock to achieve this. When done, we have obtained not only our position, but an extremely accurate time reading as well (with error on the order of 10^{-7} seconds).

With 24 satellites in the sky, maybe about 12 might be above the horizon at any time (often less in the polar regions). Some of these may be too low on the sky to be 'useable', but the orbits are designed so that at least 4 will be in fairly high positions at any time, from any point on earth. Therefore, one is always assured of being able to get a GPS positional fix. Often, 6 or 7 satellites are in good positions. To improve our readings (average out minor errors), it makes sense to try to make use of them all. So the problem of determining a fix is usually *over-determined* - we have more data than what is needed to get a unique solution.

In the next section, we will consider three numerical methods to solve this over-determined problem. As a test case, we will assume that we have six satellites (S1 - S6) and a receiver (R), located as seen in Figure 1 and described in Table 1.

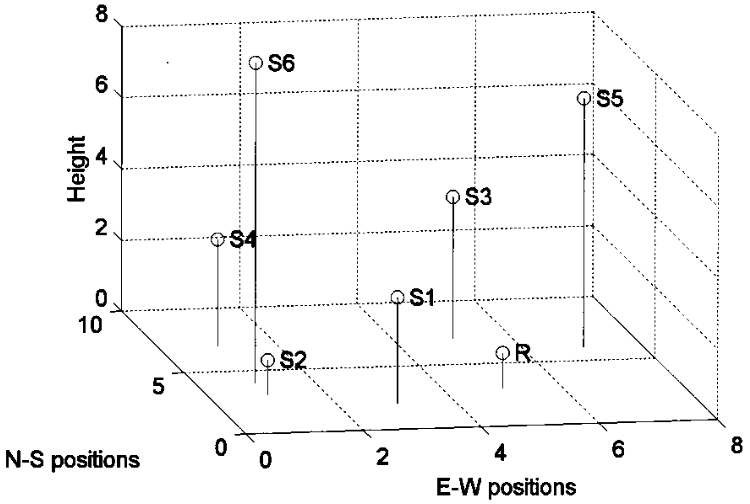

Figure 1. Location of satellites and receiver in test problem (distance units 1,000 km).

TABLE 1

Test example for algorithms to obtain receiver location

GIVEN DATA		
Transmitters (Satellites)		Recorded delay (ms) between accurate transmission time and the receive time according to inaccurate receiver clock
Nr	*x, y, z* - locations (in units of 1,000 km)	
S1	3, 2, 3	10010.00692286
S2	1, 3, 1	10013.34256381
S3	5, 7, 4	10016.67820476
S4	1, 7, 3	10020.01384571
S5	7, 6, 7	10023.34948666
S6	1, 4, 9	10030.02076857
TO BE DETERMINED (4 quantities)		
Receiver location		Clock error
R	5, 3, 1	$t = 10{,}000$

This data set is 'rigged' so that our answer (receiver position and clock error) all will be integers. This has no significance for any of the algorithms - it just it makes the equations shorter to write, and also makes it easier to follow how convergence for the algorithms is progressing.