1.1 Projectile problem

Problem:

> A projectile of mass m is shot straight up from the earth's surface with initial velocity V. Determine its subsequent height y^* as function of the time t^*. (Unrealistically), we assume that air resistance and effects due to the earth's rotation can be ignored, but *not* the decrease of gravity at increased heights.

We use stars as superscripts on y and t to indicate that these dependent and independent variables (like the parameters m and V) are expressed in units, such as meters and seconds. After formulating an ODE for $y^*(t^*)$, our next task will be to try to eliminate the dependence of units from y^* and t^*.

Governing equation in original (dimensional units):

From Newton's law Force = mass * acceleration and his law of gravity (force decaying inversely with the square of the distance) follow

$$\frac{d^2y^*}{d(t*)^2} = -\frac{gR^2}{(y^*+R)^2} \quad \text{with initial conditions} \quad y^*(0)=0,\ \frac{dy^*}{dt^*}(0)=V. \qquad (1)$$

Although equation (1) can - with some difficulty - be solved in closed form, almost any minor alteration to it (such as addition of another term) will make it unsolvable in elementary functions. Our goal in this and the next chapter is to develop a systematic, analytic approach that provides excellent approximations without depending on analytic solvability.