Chapter 4

DERIVATION AND ANALYSIS OF SOME WAVE EQUATIONS

Wave phenomena are ubiquitous in nature. Examples include water waves, sound waves, electromagnetic waves (radio waves, light, X-rays, gamma rays etc.), the waves that in quantum mechanics are found to be an alternative (and often better) description of particles, etc. Some features are common for most waves, e.g. the concept of a wave function (Section 4.1). Other features differ. In some cases, all waves travel with the same speed (e.g. sound or light in vacuum), and in other cases, the speeds depend strongly on the wavelength (e.g. water waves or particle waves). In most cases, one can start from basic physical principles and derive partial differential equations (PDEs) that govern the waves; we will do that for transverse and longitudinal waves in a string in Section 4.2. In other cases, such as water waves which are discussed in Section 4.3, the motion is not described by any single PDE. In still other cases, such as the Schrödinger equation for quantum wave functions, a very simple PDE is well known and extremely successful (describes all of chemistry!), but this equation is postulated rather than derived from simpler principles (and can be extremely difficult to solve). We note in Section 4.4 that some important linear wave equations can be formulated as systems of first order PDEs. Not only are these very well suited for numerical solution, they allow also a quite simple analysis regarding various features, such as types of waves they support and their speeds. In some cases discussed in Section 4.5, we find closed form analytical solutions. A quite general solution approach, known as the method of characteristics, is introduced in Section 4.6. This leads in Section 4.7 to the concept of Hamiltonian equations. These are fundamental in many applications, such as the study of chaotic motions. In the context of this book, their key application is to provide the governing equations for the freak wave phenomenon that is discussed in Chapter 2.