Chapter 6

THE FFT ALGORITHM

The discovery in 1965 by Cooley and Tukey of the FFT algorithm caused maybe the greatest single algorithmic revolution of all times. Applications of the FFT soon proved abundant in nearly all fields. Not only could many existing tasks be solved orders of magnitude faster, computing could be brought to bear on new areas. The FFT principle has since been been found in works of Gauss (18..) and Lanczos (19..). It is described in a numerical survey book by Runge and König (1924) and again in books on trigonometric computations by Stumpff (1937) and (1939). X-ray crysallographers in Cambridge used the method in the 1930's. It again appeared in Danielson and Lanczos (1942a,b). Still, it remained on the fringes of contemporary numerical knowledge until its revolutionary potential became apparent to the re-discoverers James W.Cooley and John W. Tukey (Tukey a professor of statistics at Princeton University, and Cooley then a programmer at IBM's Thomas J. Watson Research Center, Yorktown Heights, NY). The early history of the FFT algorithm is described in Brigham (1974).

Part of the reason the FFT idea had not impacted more earlier was that electronic computers were then not available. Clever symmetries had been found that were just as effective in speeding up the calculations for the small problem sizes that were all that then could be handled. By the end of the 1960's, big computers performed Fourier transforms over thousands of points. A dramatic example of the impact of the FFT is described by Cooley et.al. (1969) regarding the spectral analysis of a seismogram recording from the big Alaska earthquake of 1965. A 2048-point discretization of the seismic trace seen in Figure 1 a required 13½ hours to

transform on a then state-of-the-art computer (producing the spectrum seen in Figure 1 b). With the FFT algorithm, this same task took 2.4 seconds (and the result became also more accurate). The difference between the operation counts of $O(n^2)$ and $O(n \log n)$ become even larger still when n is increased further - very often the case in current applications.

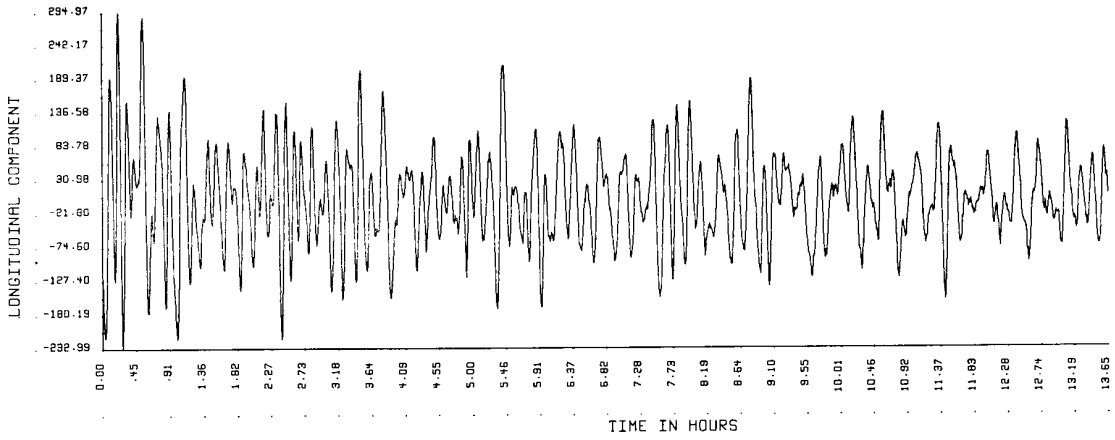

Figure 1 a. Strain seismograph of Rat Island earthquake, N = 2048, T = 13 ½ hours.

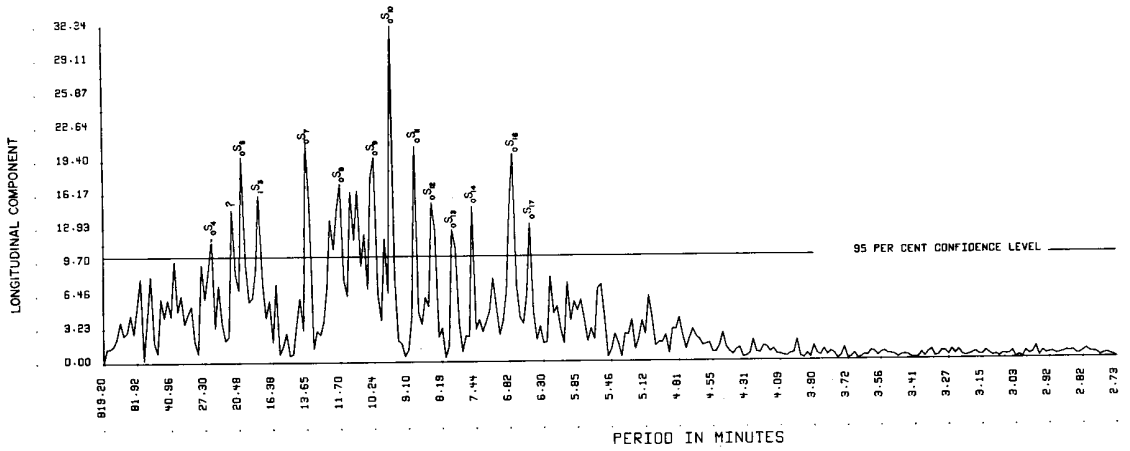

Figure 1 b. Power spectrum of the recording in Figure 1 a.
(Both Figures 1 a,b reproduced from Cooley et.al., 1969)