APPM 5560 Markov Chains

Fall 2009 Exam Two, Take Home Part

Due Monday, November 16th

Welcome to the take-home part of exam II. This is an exam, so please do not discuss it with anyone. Except me– you are more than welcome to come talk to me about it!

1. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables. Let N be a positive integer-valued random variable that is independent of the X_i sequence. We have already seen that

$$\mathsf{E}\left[\sum_{i=1}^{N} X_i\right] = \mathsf{E}[N] \cdot \mathsf{E}[X_1].$$

   Find and prove a similar formula for the variance of $\sum_{i=1}^{N} X_i$.

2. Starting at time 0, satellites are launched at times of a Poisson process with rate λ. Suppose that each satellite, once launched, has a lifetime, indepedent of all others and of the lauch process, that has cdf F and mean μ. Let $X(t)$ be the number of launched and working satellites at time t.

   (a) Find the distribution of $X(t)$.

   (b) Let $t \to \infty$ to show that the limiting distribution is Poisson($\lambda\mu$).

3. Suppose that $\{N_0(t)\}_{t\geq 0}$ is a Poisson process with rate $\lambda = 1$. Let $\lambda(t)$ denote a non-negative function of t, and let

$$\Lambda(t) = \int_0^t \lambda(s)\, ds.$$

   Define $N(t)$ by

$$N(t) = N_0(\Lambda(t)).$$

   Argue that $\{N(t)\}_{t\geq 0}$ is a non-homogeneous Poisson process with intensity (rate) function $\lambda(t)$, $t \geq 0$.