

APPM 4/5560

Solutions to Review Problems for Exam One, Part III

19.

$$\mathbf{P} = \begin{array}{c} S \quad A \quad W \\ \begin{array}{l} S \\ A \\ W \end{array} \left\| \begin{array}{ccc} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/3 & 2/3 \end{array} \right.$$

Let  $T = \min\{n \geq 0 : X_n = S\}$ .

We want to find  $E[T|X_0 = W]$ .

Let  $u_i = E[T|X_0 = i]$ . Then we want to find  $u_W$ .

Now,

$$u_W = 1 + \frac{1}{3}u_A + \frac{2}{3}u_W$$

and

$$u_A = 1 + \frac{1}{4}(0) + \frac{1}{2}u_A + \frac{1}{4}u_W.$$


---

20. Imagine the cards being laid down in “slots”. For any particular slot considered alone, you have a 1 in 13 chance of putting down the right card. So,

$$E[I_i] = P(I_i = 1) = P(\text{ith card dealt is a match}) = \frac{1}{13}$$

So,

$$E[N] = E\left[\sum_{i=1}^{13} I_i\right] = \sum_{i=1}^{13} E[I_i] = \sum_{i=1}^{13} \frac{1}{13} = 1$$


---

21. (a)

$$\begin{aligned} P(\text{2nd is A}) &= P(\text{2nd is A}|\text{1st is A})P(\text{1st is A}) + P(\text{2nd is A}|\text{1st is not A})P(\text{1st is not A}) \\ &= \frac{3}{51} \cdot \frac{4}{52} + \frac{4}{51} \cdot \frac{48}{52} \end{aligned}$$

(b)

$$\begin{aligned} P(\text{1st was A}|\text{2nd is A}) &= \frac{P(\text{2nd is A}|\text{1st was A}) \cdot P(\text{1st was A})}{P(\text{2nd is A})} \\ &= \frac{\frac{3}{51} \cdot \frac{4}{52}}{\frac{3}{51} \cdot \frac{4}{52} + \frac{4}{51} \cdot \frac{48}{52}} \end{aligned}$$