APPM 4/5560

Problem Set Eight (Due Wednesday, December 9th)

"The Last Assignment"

1. Each time a machine is repaired, it remains up and working for an exponentially distributed time with rate λ. It then fails, and its failure is either of two types. If it is a type 1 failure, then the time to repair the machine is exponentially distributed with mean μ_1; if it is a type 1 failure, then the time to repair the machine is exponentially distributed with mean μ_2. Each failure is, independently of the time it took the machine to fail, a type 1 failure with probability p and a type 2 failure with probability $1-p$.

   What proportion of time is the machine down due to a type 1 failure? What proportion of time is it down due to a type 2 failure? What proportion of time is it up?

2. Potential customers arrive at a single-server station in accordance with a Poisson process with rate λ. However, if the arrival finds n customers already in the station, then he will enter the system with probability α_n. Otherwise, he will leave. Assuming an exponential service rate μ, set this up as a birth and death process and determine the birth and death rates.

3. A small barbershop, operated by a single barber, has room for at most two customers. Potential customers arrive at a Poisson rate of three per hour, and the successive service times are independent exponential random variables with mean 1/4 hour.

   (a) What is the average number of customers in the shop?

   (b) If the barber could work twice as fast, how much more business would he do?

4. CU is overhauling it's entire telephone system. They would like to know how many phone lines they need to ensure that 99.99% of the time they have sufficient capacity to cover all outgoing calls. After extensive data collection, they estimate that the rate of calls going off campus is 74 per hour and they estimate that the mean call length is 4.2 minutes. Use an $M/M/\infty$ queue to estimate the number of phone lines needed to meet the 99.99% capacity requirement.

5. (Durrett 8.22) There are two tennis courts. Pairs of players arrive at a rate of 3 per hour and play for an exponentially distributed amount of time with mean 1 hour. If there are already two pairs of players waiting, new arrivals will leave. Find the stationary distribution for the number of courts occupied.

6. **Required for 5560 only:** After being repaired, a machine functions for an exponential time with rate λ and then fails. Upon failure, a repair process begins. The repair process proceeds sequentially through k distinct phases. First a phase 1 repair must be performed, then a phase 2, and so on. The times to complete these phases are independent, with phase i taking an exponential amount of time with rate μ_i, $i = 1, 2, \ldots, k$.

   (a) What proportion of time is the machine undergoing a phase i repair?

   (b) What proportion of time is the machine working?