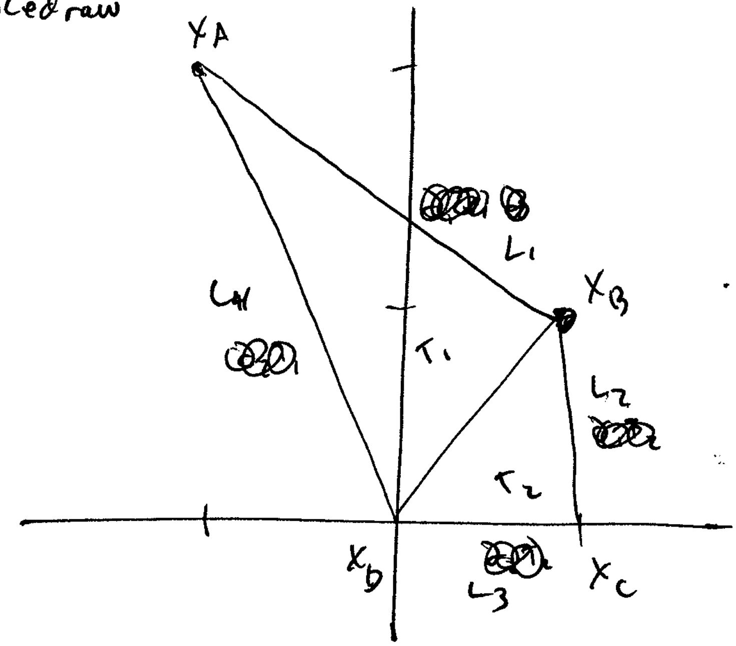
12.25

Redraw



area $T_1 \approx 1.51$

area $T_2 = \frac{1}{2}$

$$\phi_A(x,y) = \begin{cases} -\frac{x}{3} + \frac{y}{3} & T_1 \\ 0 & T_2 \end{cases} \qquad \nabla \phi_A = \begin{cases} (-\frac{1}{3}, \frac{1}{3}) \\ (0, 0) \end{cases}$$

$$\phi_B(x,y) = \begin{cases} \frac{2}{3}x + \frac{1}{3}y & T_1 \\ y & T_2 \end{cases} \qquad \nabla \phi_B = \begin{cases} (\frac{2}{3}, \frac{1}{3}) \\ (0, 1) \end{cases}$$

$$\phi_C(x,y) = \begin{cases} 0 & T_1 \\ x - y & T_2 \end{cases} \qquad \nabla \phi_C = \begin{cases} 0 \\ (1, -1) \end{cases}$$

$$\phi_D(x,y) = \begin{cases} 1 - \frac{x}{3} - \frac{2y}{3} & T_1 \\ 1 - x & T_2 \end{cases} \qquad \nabla \phi_D = \begin{cases} (-\frac{1}{3}, -\frac{2}{3}) \\ (-1, 0) \end{cases}$$

Consider $\Delta u = 0$ on $T_1 \cup T_2$

<u>BC</u>: $u(x,y) = xy$ on L_1

$$u_n(x,y) = \nabla u \cdot \hat{n} = \begin{cases} -\frac{2}{\sqrt{3}}y - \frac{1}{\sqrt{3}}x & \text{on } L_4 \\ -x & \text{on } L_3 \\ y & \text{on } L_2 \end{cases}$$

Apply BC's :

Dirichlet Part $\rightarrow$ $\gamma_A = -2$

$\gamma_B = 1$

$\left.\begin{array}{l}\gamma_C = ? \\ \gamma_D = ?\end{array}\right) \rightarrow$ to be decided by the eqn

So, multiply by ϕ_C, ϕ_D and ~~integrate~~ integrate

Use Int. By Parts to get

① $-$ ② $=$ ③ from ~~(?)~~ before

- ③ $\Delta u = 0 \Rightarrow f \equiv 0 \Rightarrow$ ③ $= 0$

- $j = c$

② $\iint_{T_1} 0 \, dx dy + \iint_{T_2} \gamma_B (0,1)\cdot(1,-1) + \gamma_C (1,-1)\cdot(1,-1) + \gamma_D(-1,0)\cdot(1,-1) \, dx dy$

$= (-\gamma_B + 2\gamma_C - \gamma_D) \iint_{T_2} dx dy$

$= (-1 + 2\gamma_C - \gamma_D) \cdot \frac{1}{2}$

① TRAP!

$$\int_{L_2} (x-y)\, \nabla(\gamma_B \phi_B + \gamma_C \phi_C)\cdot(1,0)\, ds$$

PROBLEM!

we're not using Flux info!

So,

$$\underset{(x=1)}{L_2} \Rightarrow \int_{L_2} (1-y)\boxed{\overset{\text{From Flux BC}}{y}}\, dy = \int_0^1 y - y^2\, dy = \frac{1}{2}y^2 - \frac{1}{3}y^3\Big|_0^1 = \frac{1}{6}$$

also flux

$$\underset{(y=0)}{L_3} \Rightarrow \int_{L_3} (x-0)(-x)\, dx = \int_0^1 -x^2\, dx = -\frac{1}{3}x^3\Big|_0^1 = -\frac{1}{3}$$

$$\underset{(y=-2x)}{L_4} \Rightarrow \int_{L_4} (x+(+2x))\left(\frac{+2(+2x)}{\sqrt{3}} - \frac{1}{\sqrt{3}}x\right) dx$$

$$= \int_{L_4} 3x \cdot \frac{3}{\sqrt{3}}\, x\, dx = \frac{3}{\sqrt{3}}x^3\Big|_0^1 = \frac{3}{\sqrt{3}}$$

Thus $\displaystyle\int_{L_2 \cup L_3 \cup L_4} ds = \frac{3}{\sqrt{3}} - \frac{1}{6}$

$j = D$

$$② \iint_{T_1} \left\{\gamma_A\left(-\tfrac{1}{3}, \tfrac{1}{3}\right)\cdot\left(-\tfrac{1}{3}, -\tfrac{2}{3}\right) + \gamma_B\left(\tfrac{2}{3}, \tfrac{1}{3}\right)\cdot\left(-\tfrac{1}{3}, -\tfrac{2}{3}\right) + \gamma_D\left(-\tfrac{1}{3}, -\tfrac{2}{3}\right)\left(-\tfrac{1}{3}, -\tfrac{2}{3}\right)\right\} dx\,dy$$

$$+ \iint_{T_2} \left\{\gamma_B(0,1)\cdot(-1,0) + \gamma_C(1,-1)\cdot(-1,0) + \gamma_D(-1,0)\cdot(-1,0)\right\} dx\,dy$$

$$= \left(-\tfrac{1}{9}\gamma_A + -\tfrac{4}{9}\gamma_B + \tfrac{5}{9}\gamma_D\right) 1.51 + \left(-\gamma_C + \gamma_D\right) \cdot \tfrac{1}{2}$$

①

$$L_2 \ (x=1) \rightarrow \int_{L_2} (1-(1)) \cdot y\,dy = 0$$

$$L_3 \ (y=0) \rightarrow \int_{L_3} (1-x)(-x)\,dx = \int_0^1 -x + x^2\,dx = -\frac{1}{2}x^2 + \frac{1}{3}x^3 \Big|_0^1$$

$$= -\frac{1}{6}$$

$$L_4 \ (y=-2x) \rightarrow \int_{L_4} \left(1 - \frac{x}{3} + \frac{2(+2x)}{3}\right)\left(\frac{+2}{\sqrt{3}}(+2x) - \frac{1}{\sqrt{3}}x\right) dx$$

$$= \int_{L_4} \left(1 + \frac{3}{3}x\right)\left(\frac{3}{\sqrt{3}}x\right) dx = \int_0^1 \frac{3}{\sqrt{3}}x + \frac{3}{\sqrt{3}}x^2\,dx$$

$$= \frac{3}{2\sqrt{3}}x^2 + \frac{3}{3\sqrt{3}}x^3 \Big|_0^1 = \frac{5}{2\sqrt{3}}$$

$$\int_{L_2 \cup L_3 \cup L_4} ds = \frac{-1}{6} + \frac{5}{2\sqrt{3}}$$

Thus we have

- $-\frac{1}{2} + \gamma_C - \frac{1}{2}\gamma_D + \frac{3}{\sqrt{3}} - \frac{1}{6} = 0$

- $\frac{2}{9} \cdot 1.51 - \frac{4}{9} \cdot 1.51 + \left(\frac{5 \cdot 1.51}{9} + \frac{1}{2}\right)\gamma_D - \frac{\gamma_C}{2} - \frac{1}{6} + \frac{5}{2\sqrt{3}} = 0$

$$\rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5 \cdot 1.51}{9} + \frac{1}{2} \end{bmatrix} \begin{pmatrix} \gamma_C \\ \gamma_D \end{pmatrix} = \begin{pmatrix} \frac{2}{3} - \frac{3}{\sqrt{3}} \\ \frac{2}{9} \cdot 1.51 - \frac{5}{2\sqrt{3}} + \frac{1}{6} \end{pmatrix}$$

$\therefore \ \gamma_A = -2, \ \gamma_B = +1, \ \gamma_C = -1.7421, \ \gamma_D = -1.3535$