Midterm 1

KEY

Mean 74
STDEV 13

100-80
60-80
40-60

1a) False $\|A\|_\infty = \max_{\|x\|_\infty = 1} \|Ax\|_\infty$ (5) (5)

$= \max_{\|x\|_\infty = 1} \left\| \begin{bmatrix} 10 & 15 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \right\|_\infty$

$= 25$ (5)

b. False (5) $A = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ (5)

$\rho(A) < 1$ and A is nonsingular. (5)

c. True by inequality from theorem (5)

$$\frac{\|x - x^*\|}{\|x\|} \leq \kappa(A) \frac{\|b - Ax^*\|}{\|b\|}$$ (5)

d) False. (5) we don't need specified points to construct the Fourier Series (10)

2. Yes, (5) the matrix is symmetric and (5) by Geršhgorin's theorem, (10) all evals are within a small disc around $\frac{1}{3}$ and thus > 0.

3. $\mathcal{L}_0(x) = 1$

$\mathcal{L}_1(x) = x - \frac{\langle 1, x \rangle}{\langle 1, 1 \rangle}(1) = x - \frac{\int_0^\infty x e^{-x} dx}{\int_0^\infty e^{-x} dx}$ (10)

$= x - \frac{+\left(x e^{-x}\big|_0^\infty - \int_0^\infty e^{-x} dx\right)}{\int_0^\infty e^{-x} dx} = x - \frac{0 + 0 e^{-\infty} - e^0}{e^{-\infty} - e^0} = x - 1$ (10)