APPM 5460 J.D. Meiss

Dynamics Projects

You task is to find a topic that interests you and to read about it. The final project is not intended to be original research; it is fine to present what other people have done, as long as you give them the proper credit and write your paper in your own words. The purpose is to learn, in some depth, some ideas and applications of Dynamical Systems. Your project might have a computational component, but this is not necessary. In your paper, you should explain the details of the calculations (typically a journal article will leave out many of the details). It is essential that your project deal with dynamical systems modeled by ordinary differential equations, and that it applies or goes beyond the material that we cover in class. It is also important that you not just quote from a textbook: base your project around a journal article. You may need to supplement the article with textbook material, of course, but the article should be primary.

Rules of the Game

- You should let me know about the topic and the title of your project by Feb 15. I would prefer to have at most one person working on each topic, though we can arrange to divide a topic into parts if two people are interested in it.
- A 2-4 page project proposal, with literature review will be due by March 21. The purpose of this is to get you working on the project well before it is due!
- The final project will be presented as a 20 minute presentation possibly during the last week of class or at the final exam period (May 5, 4:30-7:00PM)
- The written project will be due the last day of class (May 2).

Possible Projects

All of these projects involve some library research. Some tools that will help you:

- The MathSciNet web site http://ams.rice.edu/mathscinet/search has a very useful online reference list.
- The ISI Web of Knowledge can allow you to find modern articles that refer to a classic paper. See http://isi10.isiknowledge.com/.
- The "Frequently Asked Questions" document for the newsgroup sci.nonlinear includes a list of software and references, see http://amath.colorado.edu/faculty/jdm/faq.html.

The list below includes some suggested projects. You are by no means required to choose one of these. However any project you do choose must be approved by me.

Dynamical Systems approach to Electric Circuits

It is possible to view the equations of an electric circuit as a system of ODE's, recall §1.4. Some techniques of dynamical systems can be applied to the study of these equations. In particular, circuits for power supplies and rectifiers can be studied using Lyapunov functions and multiple-scale analysis; the goal is not always to find chaos, but in certain cases, to show stability of the system. On other cases, such as the "famous" Chua circuit, will have chaotic behavior (T. Matsumoto 1984). Some references are given in the FAQ for sci.nonlinear.

Forecasting

Nonlinear Forecasting, as introduced by Sidorovich and Farmer, is a hot topic. How does it work? Apply it to some representative data. What are the fundamental limitations on prediction implied by chaos? See e.g. Sugihara and May (G. Sugihara and R.M. May 1990).

Phase Space Reconstruction

Describe the Takens theory of phase space embedding and how it can be used in experiments to show the presence or absence of chaos. For references see (E. Ott 1994) and (A. Wolf 1985).

Dynamical Systems approach to Economics

Explain some possible applications of Dynamical Systems to Economics. In particular, discuss why some applications to Microeconomics seem to fail. Explain why the so-called "Invisible Hand" might be an idea that has no reality. This corresponds to the belief that an equilibrium allocation (zero excess demand) can be viewed as a stable fixed point solution of a suitable Dynamical System (with the introduction of a tatonnement). A good reference is Saari (D. Saari 1995).

Economic cycles

Is there an economic cycle? Why do we have periods of prosperity and periods of recession? It is not clear that economic cycles really exist (Laaksonen and Matti 1996; M. Szydowski 2001).

Models of economic growth

There are many models of economic growth, generally resulting from dynamic optimization that can be written in terms of a system of differential equations. For instance, one can study the models of Solow and Ramsey (A. Chiang 1992; R. Shone 1997; M. Klein 1998).

Chaos and Finance

Some people have proposed some interesting and controversial applications of dynamical systems to the research of capital markets. Some of the authors claim to have new points of view on how to deal with the prediction of stock prices. At the center of the controversy is the question of the validity of the Efficient Market Hypothesis, and the existence of positive Lyapunov exponents in the historical stock data. The book by Peters (E. Peters 1991) might be useful.

Kolmogorov-Arnold-Moser (KAM) Theorem

Kolmogorov proposed in 1954 a remarkable stability result for conservative (Hamiltonian) Dynamical systems. Basically, it says that such systems when weakly perturbed away from a case do not immediately become completely chaotic. This implies the possible existence, at the same time, of regular and chaotic behavior. Explain the KAM theorem, discuss its history, and the consequences for the ``ergodic hypothesis" of Boltzmann. (The proof is long and difficult, requiring lots of advanced mathematics). See (R. de la Llave 2001; J. Pöschel 2001)

Chemical Pattern Modeling

There have been interesting experiments recently on patterns arising from simple chemical reactions. Could these explain the Leopard's spots and the Zebra's stripes? See the article by Swinney (H.L. Swinney 1993) and book by Murray (J.D. Murray 1993).

Biological Modeling

Dynamical systems have been successful in Biology; recall the examples in Ch. 1. Discuss some biological models for disease propagation, population dynamics, DNA replication, etc. See the book (J.D. Murray 1993). Another interesting area is virology: (M.A. Nowak and R. May 2001).

Modeling of Chaotic Toys

Develop and investigate a mathematical model for a chaotic toy such as

- Double Pendulum (T. Shinbrot 1992).
- Magnetic Pendulum (like Wildwood pendulum in my office!).

- Sprung Pendulum (M.G. Rusbridge 1979).

Modeling the Triple Pendulum in the ITLL

The triple-pendulum chaos demonstration in the lobby of the Integrated Teaching and Learning Lab (on the east side of the engineering center) has been recently instrumented. You can take data corresponding to the position of all of the rotating joints. Your task would be to take some data and compare it to the solutions of a model for the system. Phase space embedding may prove useful (T. Sauer 1991).

Synchronization of Oscillators

Coupled oscillators provide models for diverse phenomena from Josephson junctions to the flashing of fireflies. What is remarkable is that a system with many degrees of freedom sometimes gets into a synchronized state where all the oscillators move in unison. Why does this occur and what are the requirements on the oscillators and the coupling? See (S. Strogatz and I. Stewart 1993), (N. Kopell and G.B. Ermentrout 1988) and (L. Pecora 2000).

Mixing

The problem of mixing has to do with many Industrial applications, see §1.4. How is mixing described in dynamics? Describe mixing and ergodic theory. Think about some possible industrial applications. Give a possible relation of this to Fluid Dynamics. See the book by Ottino (J.M. Ottino 1990) and (H. Aref 1984) and (I. Mezic 1999).

Poincaré's solution to the three body problem

The story of Poincaré and how he gave a solution to the Restricted Three Body Problem is the story of a contest that was fixed, so he could win it, see p. 167 of DDS and (F. Diacu and P.J. Holmes 1996). This project involves some historical research, but fortunately, two recent papers (in the Archive for History of Exact Sciences) have everything that you want to know about this interesting story.

Chaos in the Solar System

Some people claim that they have found some chaotic behavior in the solar system. What are the consequences for us? Is the claim true? What are the bases for their investigations? How did they do all this? Read papers by J. Laskar (J. Laskar 1996) and J. Wisdom (G.J. Sussman and J. Wisdom 1992).

The *N*-Body Problem

Describe some current advances in the 3,4, and 5-body problems. In particular, describe the possibility of a finite time singularity in the 5-body problem. You should show a general understanding of the *N*-body problem. See Saari and Xia (D.G. Saari and Z. Xia 1995). Another interesting area is the recent discovery of an exotic "figure-eight" orbit in the *n*-body problem, see (R. Montgomery 2001).

Central Configurations

Some special cases of the *N*-body problem are extremely regular. In fact, there are configurations that never change their relative shape. These are called central configurations and are important in the study of the general N-body problem. There are many open questions to be answered. For example, the total number central configurations for 5 bodies is not known. Understand the definition of Central Configuration and write a program that finds them for different values of the parameters. To start, you can read Moeckel (R. Moeckel 1990).

Hard vs Soft Chaos

For many years it was believed that some simple systems describing the motion of a particle in a potential well where completely chaotic, indeed Gutzwiller called these systems "hard chaos." More recently some of the systems, for example the Hamiltonian with an x^2y^2 potential have been shown to have stable periodic orbits, see (P. Dahlqvist and G. Russberg 1990). Why were these orbits missed for so long and how were they found?

Hilbert's 16th Problem

Hilbert conjectured that the number of limit cycles for a polynomial differential equation on the plane is finite. Remarkably this is still an open conjecture, though partial results have been obtained. Look up some of these results and investigate models on the computer determining the number of limit cycles numerically. Some references to this are §6.6, see also (S.L. Shi 1988) and (Y. Ilyashenko and S. Yakovenko 1995).

Control of Chaos

Chaos exhibits the surprising feature that it is quite "controllable"—indeed the sensitive dependence can be exploited to give a large effect for a small controller. A famous study by Ott, Grebogi and Yorke started this field (E. Ott 1990), but many studies have taken off from there, e.g. (M. Ding 1994; J. Starrett and R. Tagg 1995; Z.G. Li 2001).

Aref, H. (1984). "Stirring by Chaotic Advection." J. Fluid Mech. 143: 1-21.

Chiang, A. (1992). Dynamic Optimization, McGraw-Hill.

Dahlqvist, P. and G. Russberg (1990). "Existence of Stable Orbits in the X^2y^2 Potential." Physical Review Letters 65(23): 2837-2838.

de la Llave, R. (2001). A Tutorial on Kam Theory. Smooth Ergodic Theory and Its Applications (Seattle, Wa, 1999). Providence, Amer. Math. Soc. **69**: 175-292.

Diacu, F. and P. J. Holmes (1996). Celestial Encounters: The Origins of Chaos and Stablity. Princeton, Princeton Univ. Press.

Ding, M., E. Ott and C. Grebogi (1994). "Crisis Control: Preventing Chaos-Induced Capsizing of a Ship." Physical Review E 50(5): 4228-4230.

Ilyashenko, Y. and S. Yakovenko, Eds. (1995). Concerning the Hilbert 16th Problem. Providence, R.I., American Mathematical Society.

Klein, M. (1998). Mathematical Methods for Economics, Addison-Wesley.

Kopell, N. and G. B. Ermentrout (1988). "Coupled Oscillators and the Design of Central Pattern Generators." Math. Biosciences 90: 87-109.

Laaksonen and Matti (1996). "Oscillations in Some Nonlinear Economic Relationships." Chaos, Solitons and Fractals 7: 2235-2245.

Laskar, J. (1996). "Large Scale Chaos and Marginal Stability in the Solar System." Celestial Mech. Dynam. Astronom 64(1-2): 115-162.

Li, Z. G., C. Y. Wen, Y. C. Soh and W. X. Xie (2001). "The Stabilization and Synchronization of Chua's Oscillators Via Impulsive Control." IEEE Trans. Circuits Systems I Fund. Theory Appl. 48(11): 1351-1355.

Matsumoto, T., L. O. Chua and S. Tanaka (1984). "Simplest Chaotic Nonautonomous Circuit." Physical Review A 30(2): 1155-1157.

Mezic, I., S. Wiggins and D. Betz (1999). "Residence-Time Distributions for Chaotic Flows in Pipes." Chaos 9(1): 173-182.

Moeckel, R. (1990). "On Central Configurations." Math. Zeit. 205: 499-517.

Montgomery, R. (2001). "A New Solution to the Three-Body Problem." Notices of the AMS 48(5): 471-481.

Murray, J. D. (1993). Mathematical Biology. New York, Springer-Verlag.

Nowak, M. A. and R. May (2001). Virus Dynamics: Mathematical Principles of Immunology and

Virology. Oxford, Oxford University Press.
Ott, E., C. Grebogi and J. A. Yorke (1990). Controlling Chaotic Dynamical Systems. Chaos: Soviet-Americn Perspective on Noninear Science. D. K. Campbell. New York, American Inst. of Physics: 153-172.
Ott, E., T. Sauer and J. A. Yorke, Eds. (1994). Coping with Chaos: Analysis of Chaotic Data and the Exploration of Chaotic Systems. Wiley Series in Nonlinear Science. New York, John Wiley.
Ottino, J. M. (1990). Mixing, Chaotic Advection and Turbulence. Annual Reviews of Fluid Mechanics, Annual Reviews Inc. **22**: 207-53.
Pecora, L., T. Carroll, G. Johnson, D. Mar and K. S. Fink (2000). "Synchronization Stability in Coupled Oscillator Arrays: Solution for Arbitrary Configurations." Internat. J. Bifur. Chaos Appl. Sci. Engrg. 10(2): 273-290.
Peters, E. (1991). Chaos and Order in the Capital Markets. New York, Wiley.
Pöschel, J. (2001). A Lecture on the Classical Kam Theorem. Smooth Ergodic Theory and Its Applications (Seattle, Wa 1999). Providence, Amer. Math. Soc. **69**: 707-732.
Rusbridge, M. G. (1979). "Motion of the Sprung Pendulum." Am. J. Phys. 48: 146-151.
Saari, D. (1995). "Mathematical Complexity of Simple Economics." Notices Amer. Math. Soc 42(2): 222-230.
Saari, D. G. and Z. Xia (1995). "Off to Infinity in Finite Time." Notices Amer. Math. Soc. 42(5): 538-546.
Sauer, T., J. A. Yorke and M. Casdagli (1991). "Embedology." J. Stat. Phys. 65: 95-116.
Shi, S. L. (1988). "A Counterexample to a Proposed Solution of Hilbert's Sixteenth Problem for Quadratic Systems." Bull. London Math. Soc. 20(6): 597-599.
Shinbrot, T., C. Grebogi, W. J. and J. A. Yorke (1992). "Chaos in a Double Pendulum." Am. J. Phys 60: 491-499.
Shone, R. (1997). Economic Dynamics. Cambridge, Cambridge University Press.
Starrett, J. and R. Tagg (1995). "Control of a Chaotic Parametrically Driven Pendulum." Physical Review Letters 74(11): 1974-1977.
Strogatz, S. and I. Stewart (1993). "Coupled Oscillators and Biological Synchronization." Scientific American 269(December): 102-109.
Sugihara, G. and R. M. May (1990). "Nonlinear Forcasting as a Way of Distinguishing Chaos from Measurement Error in Time Series." Nature 344: 734-740.
Sussman, G. J. and J. Wisdom (1992). "Chaotic Evolution of the Solar System." Science 257(5066): 56-62.
Swinney, H. L. (1993). Spatio-Temporal Patterns: Observation and Analysis. Time Series Prediction: Forecasting the Future and Understanding the Past. A. S. Weigend and N. A. Gershenfeld. Reading, MA, Addison Wesley: 557-567.
Szydowski, M., A. Krawiec and J. Tobo (2001). "Nonlinear Oscillations in Business Cycle Model with Time Lags." Chaos, Solitons and Fractals 12: 505-517.
Wolf, A., J. B. Swift, H. L. Swinney and J. A. Vastano (1985). "Determining Lyapunov Exponents from a Time Series." Physica D 16: 285-317.