

## INTERESTING FORMULAE

### Trigonometric Identities

$$\sin \alpha = (e^{i\alpha} - e^{-i\alpha})/(2i)$$

$$\cos \alpha = (e^{i\alpha} + e^{-i\alpha})/2$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$2 \sin \alpha \cos \beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\sinh \alpha = (e^\alpha - e^{-\alpha})/2$$

$$\cosh \alpha = (e^\alpha + e^{-\alpha})/2$$

$$\cosh^2 \alpha - \sinh^2 \alpha = 1$$

$$2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$\sinh(2\alpha) = 2 \sinh \alpha \cosh \alpha$$

$$\cosh(2\alpha) = \cosh^2 \alpha + \sinh^2 \alpha$$

### Indefinite Integrals

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int x \sin(ax) \, dx = -\frac{x}{a} \cos(ax) + \frac{1}{a^2} \sin(ax)$$

$$\int x \cos(ax) \, dx = \frac{x}{a} \sin(ax) + \frac{1}{a^2} \cos(ax)$$

$$\int x^n \sin(ax) \, dx = -\frac{x^n}{a} \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) \, dx, \quad n > 0$$

$$\int x^n \cos(ax) \, dx = \frac{x^n}{a} \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) \, dx, \quad n > 0$$

$$\int \sin(ax) \cos(bx) \, dx = -\frac{1}{2} \left\{ \frac{\cos[(a-b)x]}{a-b} + \frac{\cos[(a+b)x]}{a+b} \right\}, \quad a^2 \neq b^2$$

$$\int \cos(ax) \cos(bx) \, dx = \frac{1}{2} \left\{ \frac{\sin[(a-b)x]}{a-b} + \frac{\sin[(a+b)x]}{a+b} \right\}, \quad a^2 \neq b^2$$

$$\int \sin(ax) \sin(bx) \, dx = \frac{1}{2} \left\{ \frac{\sin[(a-b)x]}{a-b} - \frac{\sin[(a+b)x]}{a+b} \right\}, \quad a^2 \neq b^2$$

$$\int \cos^2(ax) \, dx = \frac{1}{2} \left[ x + \frac{\sin(2ax)}{2a} \right]$$

$$\int \sin^2(ax) \, dx = \frac{1}{2} \left[ x - \frac{\sin(2ax)}{2a} \right]$$

$$\int e^{ax} \sin(bx) \, dx = \frac{e^{ax}}{a^2 + b^2} [a \sin(bx) - b \cos(bx)]$$

$$\int e^{ax} \cos(bx) \, dx = \frac{e^{ax}}{a^2 + b^2} [a \cos(bx) + b \sin(bx)]$$

$$\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$$

### Definite Integrals

$$\int_0^\infty e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}, \quad \operatorname{Re}(a) \geq 0$$

$$\int_0^\infty x^2 e^{-ax^2} \, dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}}, \quad \operatorname{Re}(a) > 0$$

$$\int_0^\infty \cos(px) e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-p^2/(4a)}, \quad \operatorname{Re}(a) \geq 0$$

$$\int_0^\infty \frac{\sin x}{x} \, dx = \frac{\pi}{2}$$

## Formal Fourier Facts

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right\}$$

$$a_n = \frac{1}{L} \int_{-L}^L f(z) \cos \frac{n\pi z}{L} dz, \quad b_n = \frac{1}{L} \int_{-L}^L f(z) \sin \frac{n\pi z}{L} dz$$

$$\frac{1}{L} \int_{-L}^L |f(z)|^2 dz = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} \{ |a_n|^2 + |b_n|^2 \}$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{g}(\omega) e^{-i\omega t} d\omega, \quad \hat{g}(\omega) = \int_{-\infty}^{\infty} g(t) e^{i\omega t} dt$$

## Laplacian Operator

$$\Delta u = \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}$$

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} \left( \sin \phi \frac{\partial u}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2 u}{\partial \theta^2}$$

(Spherical coordinates:  $x = r \cos \theta \sin \phi$ ,  $y = r \sin \theta \sin \phi$ ,  $z = r \cos \phi$ ).

## Div, Grad, Curl and All That

$$\nabla(a \cdot b) = (a \cdot \nabla)b + (b \cdot \nabla)a + a \times (\nabla \times b) + b \times (\nabla \times a)$$

$$\nabla \cdot (a \times b) = b \cdot (\nabla \times a) - a \cdot (\nabla \times b)$$

$$\nabla \times (a \times b) = a(\nabla \cdot b) - b(\nabla \cdot a) + (b \cdot \nabla)a - (a \cdot \nabla)b$$

$$\int_D \nabla \cdot F dx = \int_{\partial D} \hat{n} \cdot F dS$$

$$\int_R \nabla \times A \cdot \hat{n} dS = \oint_{\partial R} A \cdot dl$$

$$\int_D (v \Delta u - u \Delta v) dx = \int_{\partial D} [v(\hat{n} \cdot \nabla u) - u(\hat{n} \cdot \nabla v)] dS$$

$$\int_D \Delta u dx = \int_{\partial D} \hat{n} \cdot \nabla u dS$$

$$\int_D (v \Delta u + \nabla v \cdot \nabla u) dx = \int_{\partial D} v(\hat{n} \cdot \nabla u) dS$$