APPM 5600 NUMERICAL ANALYSIS

TEST I

TIME: 50 MINUTES September 30, 1994, 11:00–11:50 a.m.

No aids except calculators permitted

NAME:_____

For Grader Only	
1	/20
2	/10
3	/20
4	/20
5	/10
6	/20
Σ	/60

1. Write the Lagrange basis polynomials, $\{\ell_i(x)\}_{i=0}^2$, for the points $x_0 = 1, \quad x_1 = 2, \quad x_2 = 3$. Write the polynomial of degree two that interpolates $f(x) = \sinh(x)$ at the points x_0, x_1, x_2 in terms of the Lagrange basis polynomials.

2. Suppose f is a polynomial of degree k. Is the following statement true? Explain your answer.

$$f[x_0, \ldots, x_n] = 0$$

for all $n > k$.

3. Construct the natural cubic spline that interpolates $f(x) = x^4$ on the knots (nodes) $x_0 = 0$, $x_1 = 1$, $x_2 = 2$. (The natural spline has $S''(x_0) = S''(x_2) = 0$.)

4. The first two monic orthogonal polynomials with respect to the inner product

$$(\underline{f}, \underline{g}) = \int_{-1}^{1} f(x)g(x)dx$$

are $p_0(x) \equiv 1, \quad p_1(x) = x$. Compute $p_2(x)$. Then, find $q_2(x)$, the polynomial of degree two that minimizes the norm

$$\|f(x) - q_2(x)\| = \left(\int_{-1}^{1} (f(x) - q_2(x))^2 dx\right)^{\frac{1}{2}},$$

where $f(x) = e^x$. (Write down the formulas first.)

5. Consider the polynomial $p_3(x) = x^3 - \frac{1}{4}x$. Is there some weight for which this is an orthogonal polynomial on the interval [0, 1]? Explain your answer.

6. Define the inner product on $C[a,\, b]$ as

$$(f,\, g)_w = \int_a^b f(x)g(x)w(x)dx$$

for some weight function $w(x) > 0$. The modified Gram-Schmidt algorithm for constructing orthogonal polynomials can be written as

$$\begin{aligned}
p_0(x) &= 1 \\
p_1(x) &= xp_0(x) - \alpha_{0,0}p_0(x) \\
p_2(x) &= xp_1(x) - \alpha_{1,0}p_0(x) - \alpha_{1,1}p_1(x) \\
&\vdots \\
p_{k+1}(x) &= xp_k(x) - \sum_{j=0}^{k} \alpha_{k,j}p_j(x)
\end{aligned}$$

i Write a formula for $\alpha_{k,j}$

ii Show that $\alpha_{k,j} = 0$ for $j < k - 1$.