**An Example of the Alias Method**

## The Alias Method for Discrete Distributions

- One way to set up an alias table:
  - ✓ Set up vectors x, p, r and a each of length n.
    - Fill the vector x with the values $x_1, x_2, \ldots, x_n$
    - Fill the vector p with the values $p_1, p_2, \ldots, p_n$
    - Initialize the vector r with the values
      r[i]=np[i]
    - Initialize the vector a with the values
      a[i]=x[i]

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## The Alias Method for Discrete Distributions

- Define sets H and L ("high" and "low") to hold indices so that
  - $r[i] > 1 \Rightarrow i \in H$
  - $r[i] \leq 1 \Rightarrow i \in L$
- If H is empty, stop everything. This is a uniform distribution!
- If H is not empty…

## The Alias Method for Discrete Distributions

- select an index j from L and an index k from H
  - Set a[j] = k
  - Set r[k] = r[k]+r[j]-1

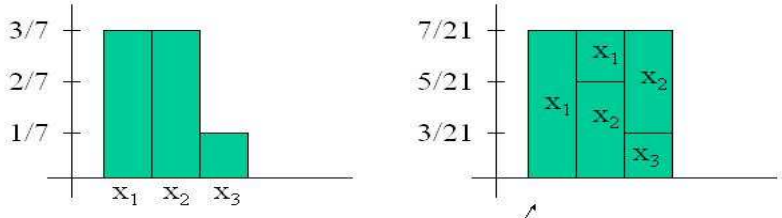
This was incorrectly written as r[j] = r[k]+r[j]-1 on the slides in class.

- If
  - r[k]<1, add k to L
  - r[k]≥1, remove k from H
- remove j from L

If H is empty, stop, otherwise, return to

## The Alias Method for Discrete Distributions

Example:




now throw "uniform darts" at this

Starting alias table for this example:

| i | x[i] | p[i] | a[i] | r[i] |
|---|---|---|---|---|
| 1 | $x_1$ | 3/7 | 1 | 9/7 |
| 2 | $x_2$ | 3/7 | 2 | 9/7 |
| 3 | $x_3$ | 1/7 | 3 | 3/7 |

Set L={3}, H={1,2}.

Since H is non-empty, we select an index j from L and k from H. Say j=3 and k=1.

Set a[3]=1, set r[1]=r[1]+r[3]-1 = 5/7

Since r[1]<1, we add 1 to L. Finally, we remove 3 from L.

The updated alias table for this example:

| i | x[i] | p[i] | a[i] | r[i] |
|---|---|---|---|---|
| 1 | $x_1$ | 3/7 | 1 | 5/7 |
| 2 | $x_2$ | 3/7 | 2 | 9/7 |
| 3 | $x_3$ | 1/7 | 1 | 3/7 |

Now L={1}, H={2}.

Since H is non-empty, we select an index j from L and k from H. Say j=1 and k=2 (not much choice here!).

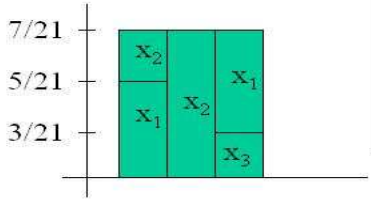
Set a[1]=2, set r[2]=r[2]+r[1]-1 = 1

Since r[2]≥1, we remove 2 from H. Finally, we remove 1 from L.

The updated alias table for this example:

| i | x[i] | p[i] | a[i] | r[i] |
|---|---|---|---|---|
| 1 | $x_1$ | 3/7 | 2 | 5/7 |
| 2 | $x_2$ | 3/7 | 2 | 1 |
| 3 | $x_3$ | 1/7 | 1 | 3/7 |

Since H is empty, we are done.




Note that this division/labeling of blocks is not unique but that with our "uniform darts" we will indeed hit $x_1$ w.p. 3/7, $x_2$ w.p. 3/7, and $x_3$ w.p. 1/7.