Take Home Exam I (Due Wednesday, October 153th)

(Note: Since this is technically an exam, you should not talk to anyone about it with the one exception that you are more than welcome and encouraged to come talk to me if you are having problems. Show all work– include all code. Have fun.)

1. (a) Program a simulation of a finite-capacity M/M/1 queue with the standard input parameters: λ, μ, and N where λ is the rate of the exponential interarrivals, μ is the rate of the exponential service times, and N is the queue capacity (including the customer in service).

   (finite capacity: If a ustomer arrives to find N people in the system, he/she will not get on line and will leave forever.)

   (b) Now, without making any alterations to your program, use it to simulate the stationary distribution for $\{X(t)\}$ for the digital telecommunication network described below with $\tau = 3$, $\delta = 2$, $B_T = 4$, $B_D = 6$. Be sure to describe what you are doing (ie: how you are mashing this situation into a finite-capacity M/M/1 queue). Provide a histogram of π.

   > Consider the following traffic control mechanism proposed for some high-speed digital telecommunication networks. It consists of two buffers: a "token" buffer of size B_T and a "data" buffer of size B_D.
   > Tokens are generated according to a Poisson process with rate τ and are stored in the token buffer. Token generated while the buffer is full are lost.
   > Data packets arrive according to a Poisson process with rate δ. If there is a token in the token buffer when a data packet arrives, it immediately removes one token from the token buffer and enters the network. If there are no tokens in the token buffer, it waits in the data buffer if there is room– if the data buffer is full, the data packet is lost.
   > Let $Y(t)$ be the number of tokens in the token buffer at time t, and let $Z(t)$ be the number of data packets in the data buffer at time t. Finally, let $X(t) = B_T - Y(t) + Z(t)$.

2. Consider an M/M/1 queue. Define the waiting time of a customer to be the *total time that customer spends in the system*, ie:

   $$\text{customer waiting time} = \text{customer delay time} + \text{customer service time}$$

   where a customer's delay time is the standing in line part of the wait. Let's use the notation

   $$W = D + S.$$

   Suppose in a simulation (in equilibrium), n customers go through the system. We could estimate the expected waiting time $\mathsf{E}[W]$ with $\hat{W} = \frac{1}{n}\sum_{i=1}^{n} w_i$, where w_i is the waiting time experienced by the i^{th} customer.

On the other hand, we could use the fact that we know that $\mathsf{E}[S] = \mu$ to give us $\mathsf{E}[W] = \mathsf{E}[D] + \mu$ and similarly use simulation to estimate the expected delay time.

Will this second method give an estimator of $\mathsf{E}[W]$ with lower variance?

Make a claim one way or the other and **either** prove it analytically **or** confirm it through simulation.

3. Two types of seeds are randomly dispersed on a one-acre field according to two independent Poisson processes with intensities λ_1 and λ_2.

   Type 1 and type 2 seeds will germinate with probabilities p_1 and p_2, respectively.

   Type 1 plants will produce K offshoot plants on runners randomly spaced around the plant where $K \sim geom(p)$. $(P(K = 0) = p)$

   Suppose that the one-acre field is evenly divided into 10x10 quadrats.

   Assume that the number of offshoot plants that fall into a quadrat different from their parent plants is negligible.

   A particular insect population can only be supported if at least 75% of the quadrats contain at least 35 plants.

   Using p=0.9, p1=0.7, and p2=0.8, explore the values of λ_1 and λ_2 that will give the insect population a 95% chance of surviving.

   (Use the hugely simplifying assumption that there is no time component to this process and, in particular, that offshoot plants do not have further offshoots.)