Contents

Population Doubling Activities

The Population Doubling project explores exponential growth, doubling time, and carrying capacities.

Opportunity 1 looks at the power of exponential growth (in this case doubling).

Opportunity 2 uses the concept of doubling time to predict the world population for different years.

Opportunity 3 explores the carrying capacity and how it is used in logistical models..

Opportunity 1

Linear change occurs when a quantity grows or decreases by the same *number* in each unit of time.
Ex: 100, 200, 300, 400, (100 change each time) or 10, 9 , 8, 7 , … decrease of 1 each time
Exponential change occurs when a quantity grows by the same *percent* in each unit of time.
Ex: 100, 200, 400, 800, (100% growth each time), 10, 9, 8.1, 7.29, 6.561, 5.9049 (10% decrease each time)

The magic penny: You find a magic penny one day. If you put it under your pillow it doubles. So you start with 1 penny, the next day you have two, the next day you have four, then eight, etc.

Day	Total Amount in $	Day	Total Amount in $
0	0.01	6	0.64
1	0.02	7	1.28
2	0.04	8	2.56
3	0.08	9	5.12
4	0.16	10	10.24
5	0.32	11	20.48

Formula for amount on day t. 0.01×2^t.

How much after 30 ($10,737,418.24days?) 1 year (7.5 x 10^{107})? How long would it take to pay off the national debt? (Approximately 11.4 trillion dollars as of June 2009) (Ans 50 days)

Ex 2: Place a single bacterium in a bottle at 11:00am. The bacterium grows and at 11:01 it divides into two bacteria. These two grow and at 11:02 divide into four bacteria, which grow into eight bacteria at 11:03. If the bacteria continue to double every minute and the bottle becomes full at 12pm, how many bacteria are in the bottle at 12pm? (ans: 2^{60}). A full bottle is bad for the bacteria because they will have run out of space. At what time is the bottle half-full? (Ans: 11:59)

How full is the bottle at 11:56am? (Ans: 1/16 full)

At 11:59 the bottle is half full. If all of the bacteria are now evenly spread out over four total bottles, what time will it be when the four bottles are full? (Ans: 12:02pm).

How Opportunity 1 is Coded

```
function calcCupcakes(event:MouseEvent):void {
	cupcakes=cupcakes*2;
	trace(cupcakes);
}
```

Opportunity 2

The Doubling Time is the amount of time it takes for something to double while undergoing exponential growth. This time is denoted T_{double}.

If the doubling time is known, the value of an exponentially growing quantity can be found by using the formula below

$$new\ value = reference\ value \times 2^{t/T_{double}}$$

where t = the time since the reference year
T_{double} = the doubling time.
For example, the world population doubled from 3 billion in 1960 to 6 billion in 2000. What is the doubling time? (Ans: 40 years). Use that doubling time to calculate the world population in 2008 (Ans: 6.9 billion).

Use a 40-year doubling time to calculate the world population for the following years in the table. Note that you can use either the 1960 or the 2000 population value for the reference value in the equation. What will change is the amount of time since the reference year. For example, 2008 is eight years past 2000 but 48 past 1960.

Year	Population (in billions)	Year	Population (in billions)
2010	Ans: 7.1	2200	Ans: 192
2020	Ans: 8.5	2500	Ans: 34,755
2050	Ans: 14.3	2525	Ans: 53,600
2100	Ans: 33.9	3000	Ans: 201,326,592*

* The value of 201,326,592,000,000,000 in 3000 illustrates one of the problems with using pure exponential growth. The equation we use calculates exponential growth forever! We will learn how to fix this in the next level.

How Opportunity 2 is Coded.

```
newvalue = (basePopulation*Math.pow(2,timet/doublingt))
newPopulation.text = newvalue.toString();
```

Opportunity 3

See the chart below for the predicted values for the world population.

Year	Value predicted using doubling time (in billions)	Value predicted by United Nations (in billions)
2008	6.9	6.7 (actual)
2010	7.1	6.8
2020	8.5	7.6
2050	14.3	9

As mentioned in Level 2, if one enters CU in 2011 and graduates in 2015, the total amount of CU tuition will be approximately $28,800. If you need to take out $14,400 in student loans, what is the monthly payment if you repay the loan in 10 years and the interest rate is 6%? What is the total amount to be paid back and how much has been paid in interest? (Ans: monthly payment= $159.87, total to be paid back = $19,184.40, interest = $4,784.40)

The equation

$$new\ value = initial\ value \times 2^{t / T_{double}}$$

tends to predict a population that is too high. This is because the world population growth is not purely exponential and the doubling time is not always 40 years.
A logistical model assumes that population growth gradually slows as the population reaches its *carrying capacity*. The carrying capacity is the largest population the environment can support for extended periods of time (think of resources such as oil, food, land, water, etc).

Accounting for a carrying capacity makes the equation more complicated.

$$P(t) = \frac{K\ P_0\ e^{rt}}{K + P_0\ (e^{rt} - 1)}$$ Where

K = The carrying capacity
P_0 = The population at a referenced year (we will use the value of 6 billion from 2000)
r = The growth rate
t = The time since the reference year (expressed in years).

For this exercise we will assume a growth rate of 2.95% and a carrying capacity of 12 billion.

Year	Value predicted using doubling time (in billions)	Value predicted by United Nations (in billions)	Logistical Model (in billions)
2008	6.9	6.7 (actual)	Ans: 6.7
2010	7.1	6.8	Ans: 6.9
2020	8.5	7.6	Ans: 7.7
2050	14.3	9	Ans: 9.8
2200	192	NA	Ans: 11.97

Now calculate the estimated populations for given K and R values.

Year	Growth Rate (K =12 billion)	Estimated Population	Year	K-Value (r = 1.2%)	Estimated Population
2008	0.5%	Ans: 6.12	2008	10	Ans: 6.22
2010	1%	Ans: 6.30	2010	15	Ans: 6.44
2020	1.2%	Ans: 6.72	2020	20	Ans: 7.05
2050	2%	Ans: 8.77	2050	5	Ans: 5.50*
2200	5%	Ans: 11.999	2200	5	Ans: 5.08*

How Opportunity 3 is Coded

```
newvalue = (carryingCap * basePopulation * Math.exp(growthRate * timetoPass)) / (carryingCap +
(basePopulation * (Math.exp(growthRate * timetoPass)-1)));

newPopulation.text = newvalue.toString();
```