GEEN 1360 (Calculus II)
Worksheet 14
April 26, 2007

Instructions: Be clear in writing up your solutions – show **all** your work and draw any necessary graphs. Approach these problems step by step, and make sure that everyone in the group can explain how the solution was found.
Website: We have a website where you can find these worksheets and their solutions:
http://amath.colorado.edu/courses/GEEN1360/2007Spr/

1. Consider the curve $x = t^3$, $y = 3t^2/2$ for $0 \le t \le \sqrt{3}$.
   (a) Compute the length of the curve. [Fall '96 Final]
   (b) Find the area of the surface generated by revolving the curve about the x-axis (integrating over t).
   (c) Set up, but **do not** evaluate the volume of revolution in Cartesian coordinates using both **disks** and **shells**.
2. Consider the region in the first quadrant bounded by $y = x - x^2$. Set up, but **do not** evaluate, the integral to find the volume obtained by revolving the region about the line $y = -2$. [Fall '99 Final]
3. Find the volume of the solid generated by revolving about the x-axis, the region in the first quadrant bounded by $y = \sin(x)$ and $x = \frac{\pi}{2}$ with $0 \le x \le \dfrac{\pi}{2}$. [Fall '99 Final]
4. Find the center of mass of the region inside $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ when $x \ge 0$ and density $\delta = 1$. [Spring '05 Final]
5. Evaluate each of the following integrals. [Spring '02 Final]
   (a) $\displaystyle\int \frac{1}{x^2 + 6x + 25}\, dx$
   (b) $\displaystyle\int \frac{1}{x^2 + 6x + 8}\, dx$
6. Explain whether the following integrals converge or diverge. [Spring '02 Final]
   (a) $\displaystyle\int_1^\infty \frac{x^2 + 1}{x^5 + x + 1} dx$
   (b) $\displaystyle\int_0^2 \frac{1}{(1-x)^2} dx$
7. Determine whether the following series converge absolutely, conditionally, or diverge. Indicate clearly **why** your answer is correct. [Fall '02 Final]
   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln(n)$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3\ln(n) + 2}{n^2 + 2n + 1}$
8. Using Maclaurin series, compute $\displaystyle\int_0^{1/10} e^{-x^2}\, dx$. Also, estimate how accurately the first four non-zero terms of the series approximate the integral's value. You may leave your answer in terms of factorials, etc. [Fall '99 Final]

9. Let $f(x) = \int_0^x \sin(t^2)\ dt$. Use the Maclaurin series for $\sin(t)$ to compute the Maclaurin series for $f(x)$. What is the minimum number of terms of the series for $f(x)$ would be needed to compute $f(0.1)$ accurately to 4 decimal places? [Fall '96 Final]

10. For what values of x do the following series converge (i) absolutely, (ii) conditionally? Justify your answers. [Fall '02 Final]

    (a) $\sum_{n=1}^{\infty} \frac{n^5(x-3)^{2n}}{4^n}$

    (b) $\sum_{n=1}^{\infty} \frac{n(\ln x)^{n-1}}{x}$

    (c) $\sum_{n=1}^{\infty} \frac{2^n x^n}{(3n)!n!}$

11. Are the following true or false? Must present answers to an LA/TA for credit [Fall '02 Final]

    (a) $x \int \frac{1}{x}\ dx = x \ln|x| + Cx$ where C is some constant.

    (b) If $\lim_{n->\infty} a_n = L$, where L is any finite value, the sequence $a_1, a_2, ...$ converges.

    (c) $\sum_{n=1}^{\infty} (e^x - 3)^n$ is a power series.

    (d) $\sum_{n=1}^{\infty} (e^x - 3)^n$ is a geometric series.