Review of Series

Examples...

Geometric Series

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \qquad \text{if} \qquad |r| < 1$$

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Variations:

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Variations:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{r-1}$$

Geometric Series

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Variations:

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(diverges otherwise)

Variations:

$$\sum_{n=2}^{\infty} r^{n-2} = \frac{1}{1-r}$$

Examples:

1

$$\sum_{n=0}^{\infty} 2\left(\frac{1}{\sqrt{2}}\right)^n$$

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$$= 2\,\frac{1}{1-\frac{1}{\sqrt{2}}}$$

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$$= 2\,\frac{1}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{2\sqrt{2}}{\sqrt{2}-1}$$

Examples:

2

$$\sum_{n=0}^{\infty} \left(\frac{2}{\sqrt{2}} \right)^n$$

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Diverges since $2/\sqrt{2} > 1$!

Examples:

3

$$\sum_{n=5}^{\infty} 3\left(\frac{2}{3}\right)^n$$

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$$\sum_{n=5}^{\infty} 3 \left(\frac{2}{3}\right)^n$$

$$= 3 \cdot \left(\frac{2}{3}\right)^5 \sum_{n=5}^{\infty} \left(\frac{2}{3}\right)^{n-5}$$

Examples:

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$$\sum_{n=5}^{\infty} 3\left(\frac{2}{3}\right)^{n}$$

$$= 3 \cdot \left(\frac{2}{3}\right)^{5} \sum_{n=5}^{\infty} \left(\frac{2}{3}\right)^{n-5}$$

$$= 3 \cdot \left(\frac{2}{3}\right)^{5} \frac{1}{1 - 2/3} = 9 \cdot \left(\frac{2}{3}\right)^{5}$$

Examples:

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$$\sum_{n=1}^{\infty} 5\frac{2^{n+1}}{3^n}$$

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$$= 5 \cdot \frac{2^2}{3} \frac{1}{1 - 2/3} = 20$$

Telescoping Series

$$\sum_{n=1}^{\infty}(b_n - b_{n+1}) = b_1$$

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...or, more general...

(... so you shouldn't use a formula)

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$$\sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

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$$S_n = \left(1 - \tfrac{1}{2}\right) + \left(\tfrac{1}{2} - \tfrac{1}{3}\right) + \left(\tfrac{1}{3} - \tfrac{1}{4}\right) + \cdots + \left(\tfrac{1}{n-1} - \tfrac{1}{n}\right) + \left(\tfrac{1}{n} - \tfrac{1}{n+1}\right)$$

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$$= 1 - \frac{1}{n+1}$$

Examples:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

So,

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n} = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(1 - \frac{1}{n+1}\right)$$

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$$= 1$$

Examples:

2

$$\sum_{n=1}^{\infty} \frac{4}{n^2 + 2n}$$

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$$S_n = \left(2 - \tfrac{2}{3}\right) + \left(1 - \tfrac{2}{4}\right) + \left(\tfrac{2}{3} - \tfrac{2}{5}\right) + \cdots + \left(\tfrac{2}{n-1} - \tfrac{2}{n+1}\right) + \left(\tfrac{2}{n} - \tfrac{2}{n+2}\right)$$

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$$= 3 - \frac{2}{n+1} - \frac{2}{n+2}$$

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$$\sum_{n=1}^{\infty} \frac{4}{n^2 + 2n}$$

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$$= 3$$

Examples:

3

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

Examples:

$\boxed{3}$

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

$$= \sum_{n=2}^{\infty} \left(\frac{1}{2(n-1)} - \frac{1}{2(n+1)} \right)$$

Examples:

3

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

$$= \sum_{n=2}^{\infty} \left(\frac{1}{2(n-1)} - \frac{1}{2(n+1)} \right)$$

$$S_n = \left(\tfrac{1}{2} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{10}\right) + \left(\tfrac{1}{8} - \tfrac{1}{12}\right) + \left(\tfrac{1}{10} - \tfrac{1}{14}\right) +$$

$$\left(\tfrac{1}{12} - \tfrac{1}{16}\right) +$$

$$\cdots + \left(\frac{1}{2(n-3)} - \frac{1}{2(n-1)}\right) + \left(\frac{1}{2(n-2)} - \frac{1}{2n}\right) + \left(\frac{1}{2(n-1)} - \frac{1}{2(n+1)}\right)$$

Examples:

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$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

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$$= \frac{3}{4} - \frac{1}{2n} - \frac{1}{2(n+1)}$$

Examples:

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

So,

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{3}{4} - \frac{1}{2n} - \frac{1}{2(n+1)} \right)$$

Examples:

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So,

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(\frac{3}{4} - \frac{1}{2n} - \frac{1}{2(n+1)} \right)$$

$$= \frac{3}{4}$$