APPM 4/5520

Solutions for Review Problems for Exam II, Sections 7.1-7.2

1. Let $u = x^2 - 5$, then $du = 2x\,dx$.

$$\int x \sec(x^2-5)\,dx = \frac{1}{2}\int 2x\sec(x^2-5)\,dx = \frac{1}{2}\int \sec\, u\,du$$

Multiplying the top and bottom by $\sec\, u + \tan\, u$, this turns into a "dw/w" problem:

$$= \frac{1}{2}\ln|\sec\, u + \tan\, u| + C = \frac{1}{2}\ln|\sec(x^2-5) + \tan(x^2-5)| + C$$

2. First, we divide:

$$\frac{4t^3 - t^2 + 16t}{t^2+4} = 4t - 1 + \frac{4}{t^2+4}$$

Then we integrate:

$$\int (4t-1)\,dt + \int \frac{4}{t^2+4}\,dt = 2t^2 - 2 + 2\tan^{-1}\left(\frac{t}{2}\right) + C$$

3.

$$1 + \cos\, x = 1 + \cos\left(2\cdot\frac{x}{2}\right) = 2\cos^2\left(\frac{x}{2}\right)$$

So, the integral becomes

$$\int \frac{2x}{2\cos^2\left(\frac{x}{2}\right)} = \frac{1}{2}\int \sec^2\left(\frac{x}{2}\right)\,dx = \tan\left(\frac{x}{2}\right) + C$$

4.

$$\int_0^{2\pi}\sqrt{\frac{1-\cos x}{2}}\,dx = \int_0^{2\pi}\sqrt{\sin^2\left(\frac{x}{2}\right)} = \int_0^{2\pi}\left|\sin\left(\frac{x}{2}\right)\right|\,dx$$
$$= \int_0^{2\pi}\sin\left(\frac{x}{2}\right)\,dx$$

(since $\sin\left(\frac{x}{2}\right) \geq 0$ on $0 \leq x \leq 2\pi$)

$$= \left[-2\cos\left(\frac{x}{2}\right)\right]_0^{2\pi} = \ldots = 4$$

5. Let $u = 2x$. Then $du = 2\,dx$ and the integral becomes

$$\frac{1}{2}\int 10^u\,du$$

Since the derivative of 10^u with respect to u is $(\ln 10)10^u$, the integral is

$$\frac{1}{2}\int 10^u\,du = \frac{1}{2}\frac{1}{\ln 10}10^u + C = \frac{10^{2x}}{2\ln 10} + C$$

6. Divide

$$\int_0^{\pi/4} \frac{1}{\cos^2 x}\,dx + \int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\,dx = \int_0^{\pi/4} \sec^2 x\,dx + \int_0^{\pi/4} \sec x \tan x\,dx$$

$$= [\tan x]_0^{\pi/4} = [\sec x]_0^{\pi/4} = [1-0] + [\sqrt{2}-1] = \sqrt{2}$$

7. Okay, this must have been a typo– it is bad form to use the variable that you are integrating with respect to at a limit of integration. However, I'll complete the problem as stated:

$$\int_\theta^2 \sin 2\theta\,d\theta = \int_\theta^2 2\sin\theta\,\cos\theta\,d\theta$$

Now let $u = \sin\theta$. Then $du = \cos\theta\,d\theta$ and the integral becomes

$$2\int_\#^\# u\,du = \left[u^2\right]_\#^\# = \left[\sin^2\theta\right]_\theta^2 = \sin^2 2 - \sin^2\theta$$

(The pound signs were used to indicate that the limits of integration for the problem when phrased in terms of "u" are different but that I did not bother to compute them– instead, I put the θ's back and used the original limits.)

8. Integration by parts. Let $u = \sin(\ln x)$ and $dv = dx$. Then $du = \cos(\ln x)\,\frac{1}{x}\,dx$ and $v = x$. Using

$$\int u\,dv = uv - \int v\,du,$$

we have

$$x\sin(\ln x) - \int x\cdot\cos(\ln x)\,\frac{1}{x}\,dx = x\sin(\ln x) - \int \cdot\cos(\ln x)\,dx$$

Now do integration by parts again with $u = \cos(\ln x)$ and $dv = dx$. Then $du = -\sin(\ln x)\cdot\frac{1}{x}$ and $v = x$. So, now we have

$$x\sin(\ln x) - \left[x\cos(\ln x) - \int x\left(-\sin(\ln x)\cdot\frac{1}{x}\right)\,dx\right]$$

$$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

So, we have that

$$\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

or

$$2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) + C$$

or

$$\int \sin(\ln x)\,dx = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$$

(different C)

9. Multiply top and bottom by e^x:

$$\int \frac{e^2\,dx}{e^{2x}+1}$$

Now let $u = e^x$. Then $du = e^x\,dx$ and the integral becomes

$$\int \frac{du}{u^2+1} = \tan^{-1}(u) = C = \tan^{-1}(e^x) + C$$

10. Split into two integrals:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

The first is simply $\sin^{-1} x$. The second integral can be done with the substitution $u = 1 - x^2$. The answer is

$$\sin^{-1} x + \sqrt{1-x^2} + C$$