

APPM 4/5520

Some Review Problems for Exam III

For questions 1-14, answer true or false. Elaborate on your answers if possible.

1. If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$ is convergent.
2. If $\sum a_n 6^n$ is convergent, then $\sum a_n (-2)^n$ is convergent.
3. If $\sum a_n x^n$ diverges when $x = 6$, then it diverges when $x = 10$.
4. The ratio test can be used to determine whether $\sum 1/n^3$ converges.
5. The ratio test can be used to determine whether $\sum 1/n!$ converges.
6. If $0 \leq a_n \leq b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

7.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$$

8. $1^x + 2^x + 3^x + \dots$ is a power series.
9. If f has infinitely many derivatives on $(-\infty, \infty)$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \text{for all } x.$$

10. If $-1 < \alpha < 1$, then $\lim_{n \rightarrow \infty} \alpha^n = 0$.
11. If $\sum a_n$ is divergent then $\sum |a_n|$.
12. If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \dots$ converges for all x , then $f'''(0) = 2$.
13. If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.

For questions 15-22, determine whether the sequence is convergent or divergent. If it is convergent, find the sum.

15. $a_n = \frac{n}{2n+5}$

19. $a_n = \sin n$

16. $a_n = 5 - (0.9)^n$

20. $a_n = (\sin n)/n$

17. $a_n = 2n + 5$

21. $\{(1 + 3/n)^{4n}\}$

18. $a_n = n/\ln n$

22. $\{(-10)^n/n!\}$

For questions 23-34, determine whether the series is convergent or divergent.

23. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

29. $\sum_{n=1}^{\infty} \frac{\sin n}{1+n^2}$

24. $\sum_{n=1}^{\infty} \frac{n+n^2}{n+n^4}$

30. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

25. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$

31. $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{5^n n!}$

26. $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

32. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$

27. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^n$

33. $\sum_{n=1}^{\infty} \frac{4^n}{n3^n}$

28. $\sum_{n=1}^{\infty} \sqrt{\frac{n-1}{n}}$

34. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$