

APPM 4/5520

Mome Review Problems for Exam III

For questions 35-38, determine whether the series is conditionally convergent, absolutely convergent, or divergent.

35. $\sum_{n=1}^{\infty} (-1)^{n-1} n^{-1/3}$

36. $\sum_{n=1}^{\infty} (-1)^{n-1} n^{-3}$

37. $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1) 3^n}{2^{2n+1}}$

38. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{\ln n}$

For questions 39-42, find the sum of the series.

39. $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n}$

40. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

41. $\sum_{n=1}^{\infty} [\tan^{-1}(n+1) - \tan^{-1} n]$

42. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{2n} n!}$

43. Express the repeating decimal $1.2345345345\dots$ as a fraction.

44. For what values of x does the series $\sum_{n=1}^{\infty} (\ln x)^n$ converge?

45. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ correct to four decimal places.

46. (a) Show that the series $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ is convergent.

(b) Deduce that $\lim_{n \rightarrow \infty} \frac{n^n}{(2n)!} = 0$.

47. Prove that if the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then the series

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n} \right) a_n$$

is also absolutely convergent.

For questions 48-49, find the radius of convergence and interval of convergence of the given series.

48. $\sum_{n=0}^{\infty} \frac{(-3)^n x^{2n}}{n+1}$

49. $\sum_{n=1}^{\infty} \frac{x^n}{3^n n^3}$

50. Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^n.$$

51. Find the Taylor series expansion of $f(x) = \sin x$ at $c = \pi/6$.

52. Find the Taylor series expansion of $f(x) = \cos x$ at $c = \pi/3$.

For questions 53-60, find the Maclaurin series for f and its radius of convergence. (Use either the direct method or the manipulation of known series. Note: You might find it easiest in some cases to use the binomial series, even though it is the subject of Section 8.11.)

53. $f(x) = \frac{x^2}{1+x}$

54. $f(x) = \sqrt{1-x^2}$

55. $f(x) = \ln(1-x)$

56. $f(x) = x e^{2x}$

57. $f(x) = \sin(x^4)$

58. $f(x) = 10^x$

59. $f(x) = 1/\sqrt[4]{16-x}$

60. $f(x) = (1-3x)^5$