APPM 4/5520

Solutions to "Mome (More!) Review Problems for Exam III"

35. The terms with absolute values form a divergent p-series ($p = 1/3$). Therefore the series is not absolutely convergent. However, without the absolute values, the series converges by the alternating series test (must show this), hence the series is conditionally convergent.

36. The terms with absolute values form a convergent p-series ($p = 3$). Therefore, the series is absolutely convergent. (Which implies that it converges without the absolute values.)

37. Use ratio test– absolutely convergent!

38. Diverges by n-th term test.

39. Geometric:

$$\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n} = 2\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n = 2 \cdot \frac{4}{5} \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^{n-1} = \frac{8}{5} \cdot \frac{1}{1-4/5} = 8$$

40. Partial fractions, telescoping, answer: $\frac{11}{18}$.

41. Write out some terms, telescoping,answer:

$$\lim_{n\to\infty} \left[\tan^{-1}(n+1) - \tan^{-1} 1\right] = \frac{\pi}{2} - \frac{\pi}{4}$$

42.

$$\sum_{n=0}^{\infty} \frac{(-x/4)^n}{n!} = e^{-x/4}$$

43.

$$1.2 + \frac{345/1000}{1-1/1000} = \frac{12}{10} + \frac{345}{9990} = \frac{4111}{3330}$$

44. Geometric, converges when $|\ln x| < 1 \Rightarrow -1 < \ln x < 1 \Rightarrow 1/e < x < e$

45. Start writing out some terms. For an alternating series, the error caused by cutting off terms is bounded by the magnitude of the first term to be cut.

$$1 - \frac{1}{32} + \frac{1}{243} - \frac{1}{1024} + \frac{1}{3125} - \frac{1}{7776} + \frac{1}{16807} - \frac{1}{32768} + \cdots$$

Since $1/32768 < 0.000031$, this term will not affect the fourth decimal place. Therefore, we may approximate the sum by

$$1 - \frac{1}{32} + \frac{1}{243} - \frac{1}{1024} + \frac{1}{3125} - \frac{1}{7776} + \frac{1}{16807} \approx 0.9721$$

46. Converges by ratio test. Therefore part (b) holds since, by the n-th term test for divergence, if that limit was not zero, the series could not converge!

47. Use the limit comparison test!

48.

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = 3x^2 < 1 \qquad \Rightarrow \qquad |x| < \frac{1}{\sqrt{3}}$$

So, the radius of convergence is $1/\sqrt{3}$. The interval of convergence is $-1/\sqrt{3} < x < 1/\sqrt{3}$ and possibly includes the endpoints which much be checked separately. The series conveges at both endpoints by the alternating series test (show!). So, the interval of convergence is $-1/\sqrt{3} \le x \le 1/\sqrt{3}$.

49. As in previous problem, apply the ratio test, get $|x|/3 < 1 \;\Rightarrow\; |x| < 3$. So, the radius of convergence is 3 and the interval of convergence includes $-3 < x < 3$. Checking the endpoints (using p-series) shows converence at both. So, the interval of convergence is $-3 \le x \le 3$.

50. Ratio test:

$$\lim_{n\to\infty}\left|x \cdot \frac{[(2n+2)(2n+1)]^2}{(n+1)^2}\right|$$