APPM 4/5520

Solutions to Problems from Chapter Six Practice Exercises

1.
$$\frac{dy}{dx} = 10 \cdot e^{-x/5} \cdot \left(-\frac{1}{5}\right) = -2e^{-x/5}$$

2.
$$\frac{dy}{dx} = \sqrt{2} \cdot e^{\sqrt{2}x} \cdot \sqrt{2} = 2e^{\sqrt{2}x}$$

3. The first term uses the product rule:
$$\frac{dy}{dx} = \frac{1}{4}\left[x \cdot e^{4x} \cdot 4 + 1 \cdot e^{4x}\right] - \frac{1}{16}e^{4x} \cdot 4 = xe^{4x}$$

4. Another product rule:
$$\frac{dy}{dx} = x^2 \cdot e^{-2/x} \cdot \frac{d}{dx}\left(-\frac{2}{x}\right) + \frac{d}{dx}\left(x^2\right) \cdot e^{-2/x}$$
$$= x^2 \cdot e^{-2/x} \cdot \frac{2}{x^2} + 2x \cdot e^{-2/x} = 2e^{-2/x} + 2xe^{-2/x}$$

5.
$$\frac{dy}{d\theta} = \frac{\frac{d}{dx}\left[sin^2\theta\right]}{sin^2\theta} = \frac{2sin\theta \cdot \frac{d}{dx}[sin\theta]}{sin^2\theta} = \frac{2sin\theta cos\theta}{sin^2\theta}$$

6.
$$\frac{dy}{d\theta} = \frac{\frac{d}{dx}\left[sec^2\theta\right]}{sec^2\theta} = \frac{2sec\theta \cdot \frac{d}{dx}[sec\theta]}{sec^2\theta} = \frac{2sec\theta \cdot sec\theta tan\theta}{sec^2\theta} = 2tan\theta$$

7. Hmm... well, we would know how to do this if this were a natural log, so let's convert it:
$$y = \log_2(x^2/2) = \frac{\ln(x^2/2)}{\ln 2}$$
Now,
$$\frac{dy}{dx} = \frac{1}{\ln 2} \cdot \frac{d}{dx}\ln(x^2/2) = \frac{1}{\ln 2} \cdot \frac{\frac{d}{dx}[x^2/2]}{x^2/2} = \frac{1}{\ln 2}\frac{x}{x^2/2} = \frac{2}{x \ln 2}$$

8.
$$y = \log_5(3x-7) = \frac{\ln(3x-7)}{\ln 5}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\ln 5} \cdot \frac{d}{dx}\ln(3x-7) = \frac{1}{\ln 5} \cdot \frac{\frac{d}{dx}[3x-7]}{3x-7} = \frac{1}{\ln 5} \cdot \frac{3}{3x-7}$$

9. Well, there's a "rule" for derivatives like this, but let's say we can't remember it. I would use "logs" to get at that exponent:

$$\ln y = -t \ln 8$$

Now the y on the left hand side is really a function of t. It's like a natural log of some function of t, so we use the chain rule and take the derivative first of the "outside log part" and then the "inside t part":

$$\frac{d}{dt} \ln y = \frac{d}{dt}(-t \ln 8)$$

$$\frac{1}{y} \cdot \frac{dy}{dt} = -\ln 8$$

$$\Rightarrow \frac{dy}{dt} = -\ln 8 \cdot y = -\ln 8 \cdot 8^{-t}$$

10. Same thing here... you either remember a rule, or you use logs to get there:

$$\ln y = 2t \cdot \ln 9$$

$$\frac{d}{dt} \ln y = \frac{d}{dt}(2t \ln 9)$$

$$\frac{1}{y} \cdot \frac{dy}{dt} = 2 \cdot \ln 9 \qquad \Rightarrow \qquad \frac{dy}{dt} = 2 \cdot y \cdot \ln 9 = 2 \cdot 9^{2t} \cdot \ln 9$$

11.

$$\frac{dy}{dx} = 5 \cdot 3.6 \cdot x^{3.6-1} = 18x^{2.6}$$

12.

$$\frac{dy}{dx} = \sqrt{2} \cdot (-\sqrt{2})x^{-\sqrt{2}-1} = -2x^{-\sqrt{2}-1}$$

13. Let's take logs to get at that exponent:

$$\ln y = \ln(x+2)^{x+2} = (x+2) \cdot \ln(x+2)$$

$$\frac{d}{dx} \ln y = \frac{d}{dx}[(x+2) \cdot \ln(x+2)]$$

(product rule on the right)

$$\frac{1}{y} \cdot \frac{dy}{dx} = (x+2) \cdot \frac{1}{x+2} + 1 \cdot \ln(x+2)$$

(Note: That numerator in $1/(x+2)$ comes from the derivative of $x+2$ with respect to x– which is 1!)

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + \ln(x+2) \qquad \Rightarrow \qquad \frac{dy}{dx} = y\,[1 + \ln(x+2)]$$

$$= (x+2)^{x+2} \cdot [1 + \ln(x+2)]$$

14. Again, take logs to get at the exponent:

$$\ln y = \ln\left[2(\ln x)^{x/2}\right] = \ln 2 + \ln\left[(\ln x)^{x/2}\right] = \ln 2 + (x/2)\ln(\ln x)$$

Now, taking the derivative of both sides with respect to x,

$$\frac{1}{y}\cdot\frac{dy}{dx} = 0 + (x/2)\cdot\frac{d}{dx}\left[\ln(\ln x)\right] + (1/2)\cdot\ln(\ln x)$$

$$= (x/2)\cdot\frac{\frac{d}{dx}\ln x}{\ln x} + (1/2)\cdot\ln(\ln x) = (x/2)\cdot\frac{1}{x\ln x} + (1/2)\cdot\ln(\ln x)$$

$$= \frac{1}{2\ln x} + \frac{\ln(\ln x)}{2}$$

So,

$$\frac{dy}{dx} = y\cdot\left[\frac{1}{2\ln x} + \frac{\ln(\ln x)}{2}\right] = 2(\ln x)^{x/2}\cdot\left[\frac{1}{2\ln x} + \frac{\ln(\ln x)}{2}\right]$$

$$= (\ln x)^{x/2}\cdot\left[\frac{1}{\ln x} + \ln(\ln x)\right]$$

15.

$$\frac{dy}{du} = \frac{1}{\sqrt{1-(\sqrt{1-u^2})^2}}\cdot\frac{d}{du}\sqrt{1-u^2} = \frac{1}{\sqrt{1-(1-u^2)}}\cdot\frac{1}{2}(1-u^2)^{-1/2}\cdot(-2u)$$

$$= \frac{1}{\sqrt{u^2}}\cdot\frac{-u}{\sqrt{1-u^2}} = \frac{1}{|u|}\cdot\frac{-u}{\sqrt{1-u^2}}$$

but since $0 < u < 1$, we have that $|u| = u$ and the naswer is then

$$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$$

16.

$$\frac{dy}{dv} = \frac{1}{\sqrt{1-(1/\sqrt{v})^2}}\cdot\frac{d}{dv}(1/\sqrt{v}) = \sqrt{\frac{v}{v-1}}\cdot\left(-\frac{1}{2v^{3/2}}\right)$$

final answer or simplify

$$\sqrt{\frac{v}{v-1}}\cdot\left(-\frac{1}{2v^{1/2}\cdot v}\right) = \frac{-1}{2v\sqrt{v-1}}$$

17.

$$\frac{dy}{dx} = \frac{\frac{d}{dx}cos^{-1}x}{cos^{-1}x} = \frac{\frac{-1}{\sqrt{1-x^2}}}{cos^{-1}x} = \frac{-1}{(cos^{-1}x)\sqrt{1-x^2}}$$

18.

$$y = z\cdot cos^{-1}z - (1-z^2)^{1/2}$$

$$\Rightarrow \frac{dy}{dz} = z\cdot\frac{-1}{\sqrt{1-z^2}} + 1\cdot cos^{-1}z - \frac{1}{2}(1-z^2)^{-1/2}\cdot(-2z) = cos^{-1}z$$

19.

$$\frac{dy}{dt} = t \cdot \frac{1}{1+t^2} + 1 \cdot tan^{-1}t - \frac{1}{2} \cdot \frac{1}{t} = \frac{t}{1+t^2} + tan^{-1}t - \frac{1}{2t}$$

20.

$$\frac{dy}{dt} = (1+t^2) \cdot \frac{d}{dt} cot^{-1}(2t) + \left[\frac{d}{dt}(1+t^2)\right] cot^{-1}(2t)$$

$$= (1+t^2) \cdot \frac{-1}{1+(2t)^2} \cdot 2 + 2t \cdot cot^{-1}(2t) = \frac{-2(1+t^2)}{1+4t^2} + 2t\, cot^{-1}(2t)$$