APPM 4/5520

Solutions to Problems from Chapter Six Practice Exercises

41. Let $u = t^2 - 25$. Then $du = 2t\,dt$ and the integral becomes

$$\int_{-25}^{-9} \frac{du}{u} = \ln|u||_{-25}^{-9} = \ln 9 - \ln 25 = \ln(9/25)$$

42. Let $u = 1 - sin\,t$. Then $du = -cos\,t\,dt$ and the integral becomes

$$-\int_{2}^{1/2} \frac{du}{u} = -\ln|u||_{2}^{1/2} = -[\ln(1/2) - \ln 2] = \ln 4$$

43. Let $u = \ln\,v$. Then $du = (1/v)\,dv$ and the integral becomes

$$\int tan(u)\,du = \int \frac{sin\,u}{cos\,u}\,du = -\ln(cos\,u) + C = -\ln(cos(\ln v)) = C$$

(The integral is one we have seen a few times in these practice problems. It is done by making another substitution, say $w = cos\,u$.)

44. Let $u = \ln\,v$. Then $du = (1/v)\,dv$ and the integral becomes

$$\int \frac{du}{u} = \ln|u| + C = \ln|\ln v| + C$$

45. Let $u = \ln x$. Then $du = (1/x)\,dx$ and the integral becomes

$$\int u^{-3}\,du = \frac{u^{-2}}{-2} + C = -\frac{1}{2u^2} + C = -\frac{1}{2(\ln\,x)^2} + C$$

46. Let $u = \ln(x-5)$. Then $du = (1/(x-5))\,dx$ and the integral becomes

$$\int u\,du = \frac{u^2}{2} + C = \frac{[\ln(x-5)]^2}{2} + C$$

47. Let $u = 1 + \ln\,r$. Then $du = (1/r)\,dr$ and the integral becomes

$$\int csc^2\,u\,du = -cot\,u + C = -cot(1 + \ln\,r) + C$$

48. Let $u = 1 - \ln\,v$. Then $du = (-1/v)\,dv$ and the integral becomes

$$-\int cosu\,du = -sin\,u + C = -sin(1 - \ln\,v) + C$$

49. Let $u = x^2$. Then $du = 2x\,dx$ and the integral becomes

$$\int x \cdot 3^{x^2}\,dx = \frac{1}{2}\int 3^{x^2} \cdot 2x\,dx = \frac{1}{2}\int 3^u\,du$$

Do you remember how to get the derivative of 3^u with respect to u?

$$y = 3^u \qquad \Rightarrow \qquad \ln y = u \cdot \ln 3 \qquad \Rightarrow \qquad \frac{1}{y}\frac{dy}{du} = \ln 3 \qquad \Rightarrow \qquad \frac{dy}{du} = y \cdot \ln 3 = 3^u \cdot \ln 3$$

So, out integral is

$$\frac{1}{2}\int 3^{x^2} \cdot 2x\,dx = \frac{1}{2} \cdot \frac{1}{\ln 3}\int 3^u \cdot \ln 3\,du = \frac{1}{2} \cdot \frac{1}{\ln 3} \cdot 3^u + C$$

$$= \frac{1}{2} \cdot \frac{1}{\ln 3} \cdot 3^{x^2} + C$$

50. Let $u = tan\,x$. Then $du = sec^2\,u\,du$ and the integral becomes

$$\int 2^u\,du$$

This can be written as (see solution to problem 49 for details)

$$\frac{1}{\ln 2}\int 2^u \cdot \ln 2\,du = \frac{1}{\ln 2}\,2^u + C = \frac{1}{\ln 2}\,2^{tan\,x} + C$$