

APPM 4/5520

Solutions to Problems from Chapter Six Practice Exercises

51.

$$\int_1^7 \frac{3}{x} dx = 3 \ln |x| \Big|_1^7 = 3(\ln 7 - \ln 1) = 3 \ln 7$$

52.

$$\int_1^{32} \frac{1}{5x} dx = \frac{1}{5} \ln |x| \Big|_1^{32} = \frac{1}{5}(\ln 32 - \ln 1) = \frac{1}{5} \ln 32$$

(Done or simplify further to $\ln 2$.)

53.

$$\int_1^4 \left(\frac{x}{8} + \frac{1}{2x} \right) dx = \frac{x^2}{16} + \frac{1}{2} \ln |x| \Big|_1^4 = \left(1 + \frac{1}{2} \ln 4 \right) - \left(\frac{1}{16} + 0 \right) = \frac{15}{16} + \ln 2$$

(or simply $15/16 + (1/2) \ln 4$)

54.

$$\begin{aligned} \int_1^8 \left(\frac{2}{3x} + \frac{8}{x^2} \right) dx &= \frac{2}{3} \ln |x| - \frac{8}{x} \Big|_1^8 = \left(\frac{2}{3} \ln(8) - 1 \right) - (0 - 8) \\ &= \frac{2}{3} \ln(8) + 7 \end{aligned}$$

Done. Can simplify further to

$$\ln(8^{2/3}) + 7 = \ln(4) + 7.$$

(Done, or simplify to $\ln 4 + 7$)

55.

$$\int_{-2}^{-1} e^{-(x+1)} dx = -e^{-(x+1)} \Big|_{-2}^{-1} = -e^0 - (-e^1) = e - 1$$

56.

$$\int_{-\ln 2}^0 e^{2w} dw = \frac{1}{2} e^{2w} \Big|_{-\ln 2}^0 = \frac{1}{2} (e^0 - e^{\ln 2^{-1}}) = \frac{1}{2} \left(1 - \frac{1}{4} \right) = \frac{3}{8}$$

57. Let $u = 3e^r + 1$. Then $du = 3e^r dr$ and the integral becomes

$$\begin{aligned} \frac{1}{3} \int_0^{\ln 5} (3e^r + 1)^{-3/2} \cdot 3e^r dr &= \frac{1}{3} \int_4^{16} u^{-3/2} du = \frac{1}{3} \frac{u^{-1/2}}{-1/2} \Big|_4^{16} = -\frac{2}{3} (16^{-1/2} - 4^{-1/2}) = -\frac{2}{3} \left(\frac{1}{4} - \frac{1}{2} \right) \\ &= -\frac{2}{3} \cdot \left(-\frac{1}{4} \right) = \frac{1}{6} \end{aligned}$$

58. Let $u = e^\theta - 1$. Then $du = e^\theta d\theta$ and the integral becomes

$$\int_0^8 u^{1/2} du = \frac{2}{3} u^{3/2} \Big|_0^8 = \frac{2}{3} (8^{3/2} - 0^{3/2}) = \frac{2}{3} \cdot 8^{3/2}$$

59. Let $u = 1 + 7 \ln x$. Then $du = \frac{7}{x} dx$ and the integral becomes

$$\frac{1}{7} \int_1^8 u^{-1/3} du = \frac{1}{7} \cdot \frac{3}{2} u^{2/3} \Big|_1^8 = \frac{3}{14} (8^{2/3} - 1^{2/3}) = \frac{3}{2} (4 - 1) = \frac{9}{2}$$

60. Let $u = \ln x$. Then $du = (1/x) dx$ and the integral becomes

$$\int_1^2 u^{-1/2} du = 2u^{1/2} \Big|_1^2 = 2(\sqrt{2} - 1)$$