**APPM 4/5520**

**Solutions to Problems from Chapter Six Practice Exercises**

61. Let $u = \ln(v+1)$. Then $du = \frac{1}{v+1}\,dv$ and the integral becomes

$$\int_{\ln 2}^{\ln 4} u^2\,du = \frac{1}{3}u^3\Big|_{\ln 2}^{\ln 4} = \frac{1}{3}\left[(\ln 4)^3 - (\ln 2)^3\right]$$

62. Let $u = t\ln t$. Then $du = \left(t\cdot\frac{1}{t} + 1\cdot\ln t\right)\,dt = (1+\ln t)\,dt$ and the integral becomes

$$\int_{2\ln 2}^{4\ln 4} u\,du = \int_{\ln 2}^{\ln 256} u\,du = \frac{1}{2}u^2\Big|_{\ln 4}^{\ln 256} = \frac{1}{2}\left[(\ln 256)^2 - (\ln 4)^2\right]$$

63. Write $\log_4\theta$ as $(\ln\theta)/(\ln 4)$:

$$\int_1^8 \frac{\log_4\theta}{\theta}\,d\theta = \int_1^8 \frac{1}{\ln 4}\frac{\ln\theta}{\theta}\,d\theta$$

Now let $u = \ln\theta$. Then $du = (1/\theta)\,d\theta$ and the integral becomes

$$\frac{1}{\ln 4}\int_0^{\ln 8} u\,du = \frac{1}{\ln 4}\cdot\frac{1}{2}u^2\Big|_0^{\ln 8} = \frac{1}{\ln 16}[(\ln 8)^2 - 0] = \frac{(\ln 8)^2}{\ln 16}$$

64. As in problem 63, we will begin by writing $\log_3\theta$ as $(\ln\theta)/(\ln 3)$:

$$\int_1^e \frac{8\ln 3\log_3\theta}{\theta}\,\theta = \int_1^e \frac{8\ln\theta}{\theta}\,\theta$$

Now let $u = \ln\theta$. Then $du = (1/\theta)\,d\theta$ and the integral becomes

$$8\int_0^1 u\,du = 4u^2\Big|_0^1 = 4$$

65. Mmm... did somebody say "inverse trig functions"? I thought so...

Let $u = 2x$. Then $du = 2\,dx$. This has the form

$$3\int_{-3/2}^{3/2}\frac{du}{\sqrt{3^2-u^2}} = 3sin^{-1}\left(\frac{u}{3}\right)\Big|_{-3/2}^{3/2} = 3\left[sin^{-1}(1/2) - sin^{-1}(-1/2)\right] = 3\left[\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right] = \pi$$

66. Let $u = 5x$. Then $du = 5\,dx$ and the integral becomes

$$\frac{6}{5}\int_{-1/5}^{1/5}\frac{5\,dx}{\sqrt{2^2-(5x)^2}} = \frac{6}{5}\int_{-1}^{1}\frac{du}{\sqrt{2^2-u^2}} = \frac{6}{5}sin^{-1}\left(\frac{u}{2}\right)\Big|_{-1}^{1}$$

$$= \frac{6}{5}\left[sin^{-1}(1/2) - sin^{-1}(-1/2)\right] = \frac{6}{5}\left[\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right] = \frac{2\pi}{5}$$

67. Let $u = \sqrt{3}t$. Then $du = \sqrt{3}\,dt$ and the integral becomes

$$\frac{3}{\sqrt{3}}\int_{-2}^{2}\frac{\sqrt{3}\,dt}{4+(\sqrt{3}t)^2} = \frac{3}{\sqrt{3}}\int_{-2\sqrt{3}}^{2\sqrt{3}}\frac{du}{2^2+(\sqrt{3}t)^2} = \frac{3}{\sqrt{3}}\,\frac{1}{2}\,tan^{-1}\left(\frac{u}{2}\right)\Bigg|_{-2\sqrt{3}}^{2\sqrt{3}}$$

$$\frac{3}{2\sqrt{3}}\left[tan^{-1}(\sqrt{3}) - tan^{-1}(-\sqrt{3})\right] = \frac{3}{2\sqrt{3}}\left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right] = \frac{\pi}{\sqrt{3}}$$

68.

$$\int_{\sqrt{3}}^{3}\frac{dt}{(\sqrt{3})^2+t^2} = \frac{1}{\sqrt{3}}\,tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\Bigg|_{\sqrt{3}}^{3}$$

$$= \frac{1}{\sqrt{3}}\left[tan^{-1}(3/\sqrt{3}) - tan^{-1}(1)\right] = \frac{1}{\sqrt{3}}\left[\frac{\pi}{3} - \frac{\pi}{4}\right] = \frac{\pi}{12\sqrt{3}}$$

69. Let $u = 2y$. Then $du = 2\,dy$ and the integral becomes

$$\int\frac{dy}{y\sqrt{(2y)^2-1^2}} = \int\frac{2\,dy}{2\,y\sqrt{(2y)^2-1^2}} = \int\frac{du}{u\sqrt{(u)^2-1^2}}$$

$$= sec^{-1}|u| + C = sec^{-1}|2y| + C$$

70.

$$24\int\frac{dy}{\sqrt{y^2-4^2}} = 24\cdot\frac{1}{4}sec^{-1}\left|\frac{y}{4}\right| + C = 6\cdot sec^{-1}\left|\frac{y}{4}\right| + C$$