

3010 Lab: One-Dimensional Dynamics

Here we will investigate some of the dynamics for one-dimensional maps using the Mac application *ID Maps*. Though you are encouraged to use this program, it would be possible to do this with other tools, and you certainly may do so if you wish.

0) Getting the Program

For the iMacs in the Claire lab and in Norlin, the app *IDMaps* is in the Applications folder. You can launch it by opening this folder (\mathbb{A} -Shift-A) and double clicking on the *IDMaps* icon

[To get the app on your own computer, you can download it from
<<http://amath.colorado.edu/faculty/jdm/1DMaps.html>>

Click on the download link for version 1.4.4. This will put a file “1Dmaps.app.” in your Downloads folder (or on your desktop). Drag the resulting application to the desktop. Double click on it to run. If you get a “this is forbidden” message, you have to “allow” an unregistered developer’s app to run. To do this, open System Preferences → Security & Privacy → General. Click on “Open anyway”, which should appear there if you just tried to run the app.]

You are expected to **hand in a report of this lab by Monday Oct 3**. The report should contain the answers to the five “tasks” below. Include some snapshots of what you’ve seen in your write-up.

1) Beginning the Exploration

- Double click on the “1Dmap” file to launch the program.

You will be presented with two windows, a “plot” window and a “text” window.

You will see the graph of the Logistic map $f(x) = \mu x(1-x)$ in red and the diagonal $y = x$ in black. The program uses μ instead of r as the parameter. Also plotted in blue is a typical cobweb diagram. The program sets $\mu = 3.569946$ initially. We’ll see why below.

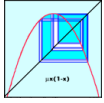
- Reset the number of iterates in the orbit to zero using the menu Parameters → Reset Orbit (equivalently \mathbb{A} -O, where \mathbb{A} is the apple or command key).

Now use the spacebar to iterate the map step by step. You can change the value of μ by using the up and down arrow keys on your keyboard (alternatively use the Parameters → Set Parameters menu item...(\mathbb{A} -K) then you must enter the values of μ and n (use 1 for now) in the parameter dialog.

Task 1: Find parameter values at which the stable period 2^1 , 2^2 , and 2^3 orbits are first created. Label these μ_1 , μ_2 , and μ_3 .

Hints:

- Use the “erase transients” command (\mathbb{A} -E) command to discard the first few iterations of your orbit. Then your orbit should eventually settle down to its asymptotic behavior.
- You can refine the parameter increment that the “arrow” keys use by using the “set parameter increment” command (\mathbb{A} -D).
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- Use the command ⌘ -Shift-4 to take a snapshot of what is on the screen, dragging the cursor over the desired region. The file, (by default a “png” file) called “Screen Shot...” will appear on the desktop. It can be inserted into your report (e.g. in MS Word use the Insert → Photo → Picture command, or simply drag the icon of the file to your document). Note: If you hit the spacebar after you press ⌘ -Shift-4, then you can take a picture of an entire window upon mouse click.
- You can resize either window by dragging from the lower right corner. The “resize box” is not drawn in the plot window, but it works nevertheless.

2) Bifurcation Diagram

That wasn't the easiest way to find the bifurcation points, and the accuracy of your values is probably not high. It is easier to use the bifurcation diagram. Create this by

- choosing “New Plot” (⌘ -N) from the File menu, and then selecting “Bifurcation Diagram” from the Show menu (⌘ -B).

The default axes are $\mu \in [0,4]$ and $x \in [-0.05,1.05]$ (see the text window). You can zoom in on any graph using the commands in the “Windows” menu. Once you select “Zoom in” (⌘ -I) the cursor changes to a magnifying glass. Drag it over the region you would like to see.

Task 2: Use this to find the bifurcation (μ) values corresponding to the first few period doublings. Record your values in a table. How high can you go?

You will note that as you do this task you need to iterate the map longer to get an accurate picture.

- Use the “Parameters → Set # Iterates” menu to increase this as you zoom in. Note: if you set it too high—you see the spinning “wait cursor” and you get tired of waiting—you can cancel the current computation by using ⌘ -. (Command period).

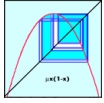
Task 3: The sequence of bifurcation values appear to be monotonically increasing, but are converging to some finite value of μ . Estimate this value.

The limit is called the “period-doubling accumulation point” and was first discovered by Feigenbaum. We'll talk more about it soon in class. If you are careful, you can compute the limit using a nice trick. Let μ_k be the parameter value at which the period 2^k orbit is created. Feigenbaum observed that

$$\mu_k = \mu_\infty + C\delta^{-k}$$

where μ_∞ is the accumulation value and δ is the famous *Feigenbaum* constant. We can estimate δ by using

$$\delta_i = \frac{\mu_i - \mu_{i-1}}{\mu_{i+1} - \mu_i}$$



Try it! To do this you should make a table something like this (the numbers below are made up):

i	μ_i	$\mu_i - \mu_{i-1}$	δ_i
1	2		
2	2.5	0.5	$0.5/0.3=1.66$
3	2.8	0.3	$0.3/0.2=1.50$
4	3.0	0.2	...

What value of δ do you get? Once you have estimated δ , you can go back to the equation for μ_i and estimate μ_∞ . What do you get?

Task 3 (extra credit): Redo the computation of the Feigenbaum constant δ for another map in the program. Select, say “Cosine” from the “Choose Map” menu, and find the sequence of period doublings. What do you get for δ ?

3) Period 3 and 5

You’ll notice that we found period 1, 2, 4, 8, 16, etc. orbits, but never period 3, 5, 7, 9, 10, etc. Where are these? As we will see later in class, *all* of these orbits *must* exist when $\mu = 4$. Therefore, they must be created somewhere. Here we’ll find period 3 and 5.

Task 4: Use the bifurcation diagram to search for the creation of period 3 and period 5 orbits. Blow up the bifurcation diagram over the region that these orbits exist. What else is going on here?

You’ll have to look in the chaotic region beyond the period-doubling accumulation, μ_∞ . Notice that this region is not uniformly filled—there are some empty regions. Inside these apparently empty regions, there is an incredibly complex bifurcation pattern!

Now go back to the cobweb picture. Select the “Set Parameters” item and type in a value of μ near the creation of the period-3 orbit. Also set $n = 3$. This means that the picture will draw f^3 instead of f . (You can also use the right and left arrow keys to increment and decrement n).

Finally set the “parameter increment” to a small value, so you can step through the region where there is a stable period three orbit.

Task 5: Show that f^3 crosses the diagonal in three new places just where the period three orbit is created. How many period three orbits are there? Which of these is stable and which is unstable? Increase the parameter value until the period three orbits are all unstable. What happens there?