2D Maps and Chaos

Lab for APPM 3010 Nov 30, 2016

Name____________________

The "Standard Map" (named by Boris Chirikov, a Russian physicist) is the 2-dimensional map

$$y' = y - \frac{k}{2\pi}\sin(2\pi x)$$
$$x' = x + y - \frac{k}{2\pi}\sin(2\pi x) \mod 1$$

As we showed in class it is a model for the *kicked rotor*. Here of course the prime, ′, means the new point after one step (*not* a derivative!). We take x mod 1, so that the line $x = -½$ and the line $x = ½$ are identified (and the phase space is wrapped into a cylinder, this can be implemented as floor(x + 0.5) – 0.5). When $k = 0$, it has simple behavior:

$$y' = y$$
$$x' = x + y \mod 1$$

So the motion moves along at constant y with x jumping by y each step, $x_t = x_o + ty$, $y_t = y_o$. Because we take x mod 1, this means that x wraps around and begins to fill in the line (a circle really) $y = const$. Our task is to investigate what happens to these orbits as k increases from 0.

The program is installed in the /Applications directory on the Macs in Claire 111. You can also download the program "StdMap" from

<http://amath.Colorado.edu/faculty/jdm/stdmap.html>

Launch the program. The program starts at $k = 0.971$ and iterates random initial conditions. You can iterate initial conditions by clicking in the main graphics window. You can change the parameter k either by using the *Change* → *Map Parameters* Menu (⌘-K), or the *up* and *down* arrow keys.

a) Set $k = 0$. Choose the *Find* → *Single Step* menu. Click on a point in the plot window to start iteration. Hit the space bar repeatedly to iterate. What happens to the iterates? What happens when x increases to ½?

b) Choose a low-order rational value of y, such as $y = \frac{2}{5}$, for an initial condition, using the *Find* → *Set Initial Condition* menu. What is different?

Now switch to *Find* → *Continuously Iterate* (⌘-G). Your initial point will rapidly iterate (as a pixel sized point), and each time you click in the plot window, you'll get a new initial condition and a new orbit.

c) Now we will investigate the orbits as k increases Using the *up arrow* key, increase k and click various places in the window to see the orbits. For what value of k does *chaos* first appear? Where in the (x,y) plane does it first appear?

d) Increase k up to about 6. When does the <u>entire</u> phase space first look like it becomes chaotic?

e) Now set $k = 2\pi$. (Use the *Change* → *Map Parameters* menu). Increase k a small amount from this point. What happens?

f) Change k back near 1 or 2. Find the stable and unstable manifolds of the fixed point by using the *Find* → *Stable Manifold* (⌘-H) menu. The default values in the dialog (0,1,±1) that appears are fine (try changing the sign!). Iterate the manifolds by hitting "space", or clicking the button. The *red* curve is the unstable manifold and the *blue* curve is the stable manifold. The manifolds begin at the fixed point $(x,y) = (0.5,0)$. What type of fixed point is it? (When you are done with a manifold, click the "Stop" button, or type "s").

g) What happens to the manifolds as you iterate them? By varying k compare this with the chaos threshold you found in (c).

h) When k is small the fixed point at (0,0) is a center. At what value of k does it becomes unstable? At this point a period-two orbit is created. Follow these new orbits until they lose stability (at what value of k?). (Select *Change*→*Clipping*→ *0<y<1* to be able to see what happens near $y = \frac{1}{2}$. Change the window bounds to 0<y<1 with the *Window*→*Choose Plot Range* command). What happens then?