Projects for Applied Math 3010: Fall 2016

You should choose a project by Sept 12. Please come to my office to discuss your choice. Proposals, in the form of 1-2 page description of project with some references are due Nov 2. The projects will be presented in class (20 minute presentation) during the last week of class and the final exam period. The written project is due the last day of class, Dec 9.

Below I have some brief descriptions of some possible projects. You are by no means limited to these; however, your topic must be approved by me. Please come discuss with me your own ideas, or let me know which of the ideas below you will do.

Your grade will reflect your ability to synthesize material from a number of sources—do not simply copy from a single book or article. Your project probably will, but need not, involve computer investigations, either in the form of a program you write, or the use of an existing program such as Matlab or Maple. Your project may involve constructing a physical model of a chaotic system, or analysis of the data of such a system (such as the stock market).

1. Phase Space Reconstruction

Describe the Takens' theory of phase space embedding and how it can be used in experiments to show the presence or absence of chaos. For references Ott, E., T. Sauer, et al., Eds. (1994). Coping with Chaos: Analysis of Chaotic Data and the Exploration of Chaotic Systems. Wiley Series in Nonlinear Science. New York, John Wiley. and Wolf, A., J. B. Swift, et al. (1985). "Determining Lyapunov exponents from a time series." Physica D **16**: 285-317.

2. Forecasting

Nonlinear Forecasting, as introduced by Sidorovich and Farmer, is a hot topic. How does it work? Apply it to some representative data. What are the fundamental limitations on prediction implied by chaos? Some data for trials is available at <http://robjhyndman.com/TSDL/>. See e.g. Sugihara, G. and R. M. May (1990). "Nonlinear Forcasting as a Way of Distinguishing Chaos from Measurement Error in Time Series." Nature **344**: 734-740.

3. Chaotic Codes

Pecora and collaborators have proposed synchronization of chaotic circuits has a technique for coding information. How does this work? Write a program to try it out. See Pecora, L. M. and T. L. Carroll (1991). "Driving systems with chaotic signals." Phys. Rev. A **44**: 2374-2383.

4. Unimodal Maps

Discuss what the Sharkovski sequence (the order in which periodic orbits occur) and the kneading theory tell us about the onset of chaos for unimodal, one-dimensional maps (see Devanay's book). Investigate how this changes if some assumptions (e.g. about the derivative of the map) are not satisfied.

5. Chaotic Scattering

Scattering is a problem with many applications. The fate of a particle, ray of light, etc. after interacting with a scatterer is investigated. This fate often depends sensitively on the initial conditions, resulting in fractal behavior. See for example a fractal created with Christmas balls http://dx.doi.org/10.1038/20573 Research chaotic scattering, Wada basins of attraction, and create your own chaotic scatterers.

6. Dynamics on Networks

Complex networks are a hot topic. See for example the review M. E. J. Newman, *SIAM Review* **45**, 167-256 (2003). http://aps.arxiv.org/abs/cond-mat/0303516/. Investigate examples of networks of coupled dynamical systems. Or better, pick a phenomenon taking place in a network (i.e, friendship dynamics on facebook, epidemics outbreak, etc) and try to construct a mathematical model for it. Study it numerically and/or theoretically.

7. Diffusion Limited Aggregation

The process of particles diffusing and sticking to each other to form clusters is described as "Diffusion Limited Aggregation", and results in striking patterns that are observed in bacterial colonies and mineral deposits. See <http://en.wikipedia.org/wiki/Diffusion-limited_aggregation>. Investigate the mathematical modeling of DLA and explore it numerically through computer simulations.

8. Catastrophe Theory

Investigate the Catastrophe Theory Controversy between Smale, Thom and Zeeman (see me to get Zeeman's recent lecture on this). Of course, you should discuss catastrophe theory itself as well.

9. Hilbert's 16th Problem

Hilbert conjectured that the number of limit cycles for a polynomial differential equation on the plane is finite. Remarkably this is still an open conjecture, though partial results have been obtained. Lookup up some of these results. Investigate models on the computer determining the number of limit cycles numerically.

10. Symbolic Programming

Write Mathematica or Maple procedures to construct bifurcation diagrams and/or periodic orbit finding routines. Apply these to one and/or two dimensional maps.

11. Complex Dynamics

Discuss Julia Sets and the Mandelbrot Set. Do much more than copy a program for making pictures of these sets—any high school student can do this! See e.g. Peitgen, H. O., H. Jürgens, et al. (1992). Chaos and Fractals. New York, Spinger-Verlag.

12. Newton's Method

Show how Newton's method, in the complex domain, can have chaotic behavior. Investigate this method, or other root finders for some example functions. See Benziger, H. E., S. A. Burns, et al. (1987). "Chaotic complex dynamics and Newton's method." Phys. Lett. A **119**: 442.

13. Fractal Dimension

What are the definitions of dimension for fractals? Compute the fractal dimension using some of these techniques for some chaotic systems. Investigate the "$f(\alpha)$" statistics. See Eckmann, J.-P. and D. Ruelle (1992). "Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems." Physica D **56**: 185-187.

14. Circle Maps

Discuss Arnold's circle map $\theta \to \theta + \omega + k \sin(2\pi\theta)$. Interesting phenomena include mode locking intervals or tongues, and the universality near the point k=1 for the golden mean frequency. See Ch. 4 of Strogatz and Bak, P., T. Bohr, et al. (1985). "Mode-locking and the transition to chaos in dissipative systems." Physica Scripta **9**: 50.

15. Hamiltonian Chaos

Discuss the transition to chaos for an area preserving map, such as the standard map. Investigate the Chirikov "resonance overlap" criterion. See MacKay and Meiss for relevant references. My Macintosh program StdMap might be helpful. See Meiss, J. D. (1992). "Symplectic Maps, Variational Principles, and Transport." Reviews of Modern Physics **64**(3): 795-848.

16. Chaos in Sudoku

An algorithm for solving a Sudoku can be formulated as a dynamical system, and this can be transiently chaotic. See http://arxiv.org/pdf/1208.0370v1.pdf "The Chaos within Sudoku," by Ercsey-Ravasz. You might also investigate other games for chaos.

17. NonSmooth Dynamics

Some dynamical systems have "impacts" and are modeled by differential equations or maps that are not continuous. Examples include sliding blocks with static/dynamic friction and buck-boost power converters. A book on this subject is di Bernardo, M., C. J. Budd, A. R. Champneys and P. Kowalczyk (2008). Piecewise Smooth Dynamical Systems. Theory and Applications. London, Springer-Verlag. Buck-boost power converters are studied in Zhusubaliyev, Z. T. and E. Mosekilde (2006). "Torus Birth bifurcations in a DC/DC Converter." IEEE Trans. Circuits and Systems **53**(8): 1839-1850.

18. Chemical Patterns Models

There have been interesting experiments recently on patterns arising from simple chemical reactions. Could theses explain the Leopard's spots and the Zeebra's stripes? See Swinney, H. L. (1993). Spatio-temporal patterns: Observation and analysis. Time series prediction: Forecasting the future and understanding the past. A. S. Weigend and N. A. Gershenfeld. Reading, MA, Addison Wesley**:** 557-567.

19. Biological Modeling

Discuss some biological models for synchronization (fireflies), population dynamics, etc. See the book Mathematical Biology by Murray for possibilities.

20. Cardiac Dynamics

Is there chaos in the heart? Period doubling bifurcations are important in cardiac dynamics. Investigate their role in cardiac arrhythmias. For an introduction to the topic, see "Nonlinear Dynamics of Heart Rhythm Disorders", Phys. Today **60** March 51 (2007). http://dx.doi.org/10.1063/1.2718757

21. Swarming Dynamics

Impressive collective behavior can be accomplished by swarms of individuals following simple dynamical rules. See for example Iain Couzin's webpage <http://icouzin.princeton.edu/> Investigate what is known about the rules that fish, ants, locusts, etc. follow and why they result in such a highly coordinated behavior.

22. Modeling of Chaotic Toys

Develop and investigate a mathematical +computational model for a chaotic toy such as

a) Double Pendulum
b) Magnetic Pendulum
c) Rattleback
d) Sprung Pendulum
e) or develop your own example

23. Nonlinear Circuits

Build a nonlinear oscillator to demonstrate the Period doubling route to chaos.

24. The 3*x*+1 problem (Collatz Problem)

The behavior of the simple map $x' = 3x+1$ (if x is odd) and $x' = x/2$ (if x is even) on the integers is conjectured to be very simple (every orbit is attracted to the periodic cycle $4 \to 2 \to 1 \to 4$...). However this has never been proved! See J. Lagarias "The 3*x*+1 Problem and its Generalizations," Amer. Math. Monthly, **92** (1985), and the video <https://youtu.be/5mFpVDpKX70>.