Problem 1 (Extrapolation)

Suppose \( Y(s) = \mu + Z(s) \) where \( Z(s) \) is a mean zero isotropic process with covariance function \( C(r) \). Suppose we have only one observation \( Y(0) = y \) at the origin, and wish to predict at \( s_0 \neq 0 \). Find the kriging predictor \( \hat{Y}(s_0) \) as \( \|s_0 - 0\| \to \infty \) for the cases

(a) \( \mu \) is known (i.e., simple kriging)

(b) \( \mu \) is unknown (i.e., ordinary kriging).

In other words, find the value that \( \hat{Y}(s_0) \) tends toward, and also find the limiting predictive variance in both cases.

Problem 2 (Building intuition)

Simulate two mean zero isotropic processes \( Z(t), t \in [0, 20] \) at a set of 15 locations under:

(a) equally spaced observation locations and

(b) unequally spaced observation locations.

For each of these two datasets, use simple kriging under three different choices of covariances to predict the process at 1000 equally-spaced locations in \([0,20]\), along with 95% pointwise confidence intervals.

Report your choice of covariance models you used to krige, plot all predictions and predictive intervals, and comment on the differences.

Problem 3 (Remote sensing temperature estimates)

The SatelliteExample.RData file on the course website contains longitudes, latitudes and temperature level-3 estimates from the MODIS satellite constellation. The two temperature variables are \( T \) and \( T.test \). Your goal is to develop and estimate a spatial model based on \( T, lon \) and \( lat \), and krige to the test data locations (\( T.test, lon.test \) and \( lat.test \)).

(a) Based on only the training data, provide a binned semivariogram. Does it look like there is evidence of a nugget effect?
(b) If you believe there is a mean trend, remove it and plot a binned semivariogram based on the detrended data. Is there evidence of a nugget effect?

(c) Decide on an appropriate statistical model for the training data. Estimate any spatial parameters using the empirical semivariogram. Include the fitted line on the binned empirical semivariogram.

(d) Krige the training data to the testing locations and produce a quilt plot (similar to below) based on both the kriged data and also on the held-out true data (T.test). How well do your kriged estimates reproduce the overall structure in the held-out data?

(e) At each prediction location in the test data, you have a predictive uncertainty based on your model. Calculate the coverage percentage of the 95% prediction interval – do these intervals contain the truth about 95% of the time?

(f) Produce a gridded map of predicted temperatures, along with associated predictive standard errors.

Figure 1: MODIS estimates of temperature; grey dots are prediction locations.