Homework set 4 — APPM5450, Spring 2006

The problems in the text-book are excellent. Do as many of the problems 9.1 – 9.11 as you have time for. If you don't have time to look at all, then I would recommend that you do these first: 9.1, 9.5, 9.7, 9.8, and 9.10.

Some comments on the problems:

9.1: Easy.

9.2: Requires a little more work than one might think. Compare Prop 9.12.

9.3: You may find the identity $A^{-1} - B^{-1} = A^{-1}(B - A)B^{-1}$ useful. Prove that from the resolvent equation, it follows that

$$\lim_{\mu \to \lambda} \frac{R_\lambda - R_\mu}{\lambda - \mu} = -R_\lambda^2.$$

(Using the textbook's sign in the definition of a resolvent.)

9.4: ...

9.5: Note that any "non-negative" operator is implicitly assumed to be self-adjoint.

9.6: ...

9.7: For (c), use formula (9.5). By partial integration, you can show that $||A^n|| \to 0$. For (d), show that 0 cannot be an eigenvalue by rewriting the integral equation as an ODE. (Similar problems have occured on the analysis prelims.)

9.8: This problem is very easily solved by working in the Fourier domain.

9.9: Again, work in the Fourier domain.

9.10: A good example of an operator with a residual spectrum (note that it is not a normal operator).

9.11: From the statement proved here, a very important fact follows: If $\lambda \in \sigma_c(A)$, then there exists a sequence of vectors $(x_n)_{n=1}^\infty$ such that $||x_n|| = 1$ and $||(A - \lambda I)x_n|| \to 0$.