

### Homework set 1 — APPM5450

From the textbook: 7.1, 7.3, 7.4. Optional: 7.2, 7.6.

**Problem 1:** Suppose that  $H$  is a Hilbert space, and that  $(\psi_n)_{n=1}^\infty$  is an ON-set in  $H$ . Let  $\mathcal{P}$  denote the set of finite linear combinations of elements in  $(\psi_n)_{n=1}^\infty$ . Prove that:

$$(\psi_n)_{n=1}^\infty \text{ is a basis for } H \iff \mathcal{P} \text{ is dense in } H.$$

**Problem 2:** Suppose that  $f, g \in C(\mathbb{T})$ . Prove that:

(a)  $f * g \in C(\mathbb{T})$ .

(b)  $f * g = g * f$ .