Homework set 7 — APPM5450, Spring 2007

From the text-book: 9.19, 9.20, 9.22, 11.1(b,c), 11.2. Optional: 9.21.

Problem: Consider the linea space $L = \mathbb{R}^2$. Define for $x = (x_1, x_2) \in L$ the seminorms

$$p_1(x) = |x_1|, \qquad p_2(x) = |x_2|.$$

Construct for $x \in L$, $j \in \{1, 2\}$, and $\varepsilon \in (0, \infty)$, the sets

$$\mathcal{B}_{x,j,\varepsilon} = \{y \in L : \, p_j(x - y) < \varepsilon\}.$$

Describe these sets geometrically. What is the topology generated by the collection of semi-norms $\{p_1\}$? Is it Hausdorff? What is the topology generated by the collection of semi-norms $\{p_1, p_2\}$? Is it Hausdorff?