Homework 4, Math222a:

**Problem 1:** Let $A$ be an $m \times n$ matrix of rank $k$. Provided that $m, n, k$ are as in the top row, please answer questions 1 through 6 in the table below.

<table>
<thead>
<tr>
<th>$m &gt; n &gt; k$</th>
<th>$m &gt; n = k$</th>
<th>$m = n = k$</th>
<th>$m &gt; n &gt; k$</th>
<th>$n &gt; m = k$</th>
<th>$n &gt; m &gt; k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q3:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q4:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q5:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q6:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Q1:** Is it the case that $\text{span}(A) = \mathbb{R}^m$?
- **Q2:** Is it the case that $A\mathbf{x} = \mathbf{b}$ always has a solution?
- **Q3:** Is the map $\mathbf{x} \mapsto A\mathbf{x}$ onto?
- **Q4:** Are the columns of $A$ linearly independent?
- **Q5:** Provided that $A\mathbf{x} = \mathbf{b}$ is consistent, is the solution unique?
- **Q6:** Is the map $\mathbf{x} \mapsto A\mathbf{x}$ one-to-one?

**Problem 2:** Consider the map

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}.$$

- Is $T$ linear?
- If $T$ is linear, what is its standard matrix?
- Is $T$ onto?
- Is $T$ one-to-one?
- Make two sketches similar to Fig. 6 in ch. 1.8 showing the geometric action of $T$.

Also do:
Section 1.6: 6, 8
Section 1.8: 6, 20, 26, 32
Section 1.9: 6, 8, 10, 24, 30, 36