Projects for Applied Math 3010: Fall 2005

You should choose a project by Sept 19. Please come to my office to discuss your choice. Proposals, in the form of 1-2 page description of project with some references are due Oct 13. The projects will be presented in class (20 minute presentation) during the last week of class and the final exam period. The written project is due the week of Dec 5. Below I have some brief descriptions of some possible projects. You are by no means limited to these; however, your topic must be approved by me. I prefer that each student chooses a different topic, so that we don't have repetition during the presentations. Please come discuss with me these topics or your own ideas.

Your grade will reflect your ability to synthesize material from a number of sources—do not simply copy from a single book or article. Your project may, but need not, involve computer investigations, either in the form of a program you write, or the use of an existing program. Your project can involve constructing a physical model of a chaotic system, or analysis of the data of such a system (such as the stock market).

1) Phase Space Reconstruction

Describe the Takens theory of phase space embedding and how it can be used in experiments to show the presence or absence of chaos. For references Ott, E., T. Sauer, et al., Eds. (1994). Coping with Chaos: Analysis of Chaotic Data and the Exploration of Chaotic Systems. Wiley Series in Nonlinear Science. New York, John Wiley. and Wolf, A., et al. (1985). "Determining Lyapunov exponents from a time series." Physica D 16: 285-317.

2) Forecasting

Nonlinear Forecasting is a hot topic. How does it work? What are the fundamental limitations on prediction implied by chaos? Some trial data for is available at http://www-personal.buseco.monash.edu.au/~hyndman/TSDL/index.htm. See Sugihara, G. and R. M. May (1990). "Nonlinear Forecasting" Nature 344: 734-740.

3) Chaotic Codes

Pecora and collaborators have proposed synchronization of chaotic circuits has a technique for coding information. How does this work? Write a program to try it out. See Pecora, L. M. and T. L. Carroll (1991). "Driving systems with chaotic signals." Phys. Rev. A 44: 2374-2383. Similar ideas are in Strogatz, Section 9.6

4) Unimodal Maps

Discuss what the Sharkovski sequence (the order in which periodic orbits occur) and the kneading theory tell us about the onset of chaos for unimodal, one-dimensional maps (see Devaney's book). Investigate how this changes if some assumptions (e.g. about the derivative of the map) are not satisfied.

5) Poincaré and the n-body Problem

Poincaré was given a prize by King Oscar of Sweden for finding solutions to the "restricted three body problem." It turns out that this contest was fixed, and that his solution to the problem was incorrect! See Andersson, K. G. (1994), "Poincaré's Discovery of Homoclinic Points," Arch. Hist. Exact Sci. 48, 133-147 and Barrow-Green, J. (1997), *Poincaré and the Three Body Problem* (Amer. Math. Soc., London).

6) Complex Dynamics

Discuss Julia Sets and the Mandelbrot Set. Do much more than copy a program for making pictures of these sets—any high school student can do this! See e.g. Peitgen, H. O., H. Jürgens, et al. (1992). Chaos and Fractals. New York, Springer-Verlag.

7) Newton's Method
Show how Newton's method and its generalizations can have chaotic behavior in the complex domain. Investigate this method, or other root finders for some example functions. See Benziger, H. E., S. A. Burns, et al. (1987). "Chaotic complex dynamics and Newton's method." Phys. Lett. A 119: 442.

8) Fractal Dimension
While we will define the box counting dimension in class, there are a number of other fractal dimensions used. Compute the fractal dimension using some of these techniques for some chaotic systems. Investigate the "$f(\alpha)$" statistics. See Eckmann, J.-P. and D. Ruelle (1992). "Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems." Physica D 56: 185-187.

9) Circle Maps
Discuss Arnold's circle map $\theta \to \theta + \omega + k\sin(2\pi\theta)$. Interesting phenomena include mode locking intervals or tongues, and the universality near the point k=1 for the golden mean frequency. See Chapter 4 of Strogatz and Bak, P., T. Bohr, et al. (1985). Physica Scripta 9: 50.

10) Controlling Chaos
How can one control chaos, which by its very definition is virtually unpredictable? In 1990, Ott, Grebogi and Yorke (OGY) came up with a scheme for doing this. Many papers have been written on this in the past 10 years, see <http://amath/faculty/jdm/faq-[3].html – Heading27>

11) Hamiltonian Chaos
Discuss the transition to chaos for an area preserving map, such as the standard map. Investigate the Chirikov "resonance overlap" criterion. See MacKay and Meiss for relevant references. The program StdMap <ftp://amath.Colorado.edu/pub/dynamics/programs> for the Macintosh might be helpful. See Meiss, J. D. (1992). "Symplectic Maps, Variational Principles, and Transport." Reviews of Modern Physics 64(3): 795-848.

12) Chaotic Spacecraft Orbits
In these days of smaller funding, NASA has been forced to use more complex orbits to get spacecraft to their targets. The first such orbit was for the Jacobini-Zinner comet mission. For example, the spacecraft Genesis, had a complex orbit that ultimately crash landed on the earth <http://genesismission.jpl.nasa.gov/>. There are many techniques to design such orbits; one was given in Bollt, E. and J. D. Meiss (1995), "Targeting Chaotic Orbits to the Moon Through Recurrence," Phys. Lett. A 204, 373-378.

13) Chemical Patterns
Patterns can arise from simple chemical reactions. Could theses explain the Leopard's spots and the Zebra's stripes? See Swinney, H. L. (1993). Spatio-temporal patterns: Observation and analysis. in Time series prediction: Forecasting the future and understanding the past. (Addison Wesley) pp557-567.

14) Biological Modeling
Discuss some biological models for synchronization (fireflies), population dynamics, etc. See the book Murray, J. (1993), Mathematical Biology (Springer-Verlag, New York).

15) Modeling of Chaotic Toys
Develop and investigate a mathematical +computational model for a chaotic toy such as
Double Pendulum <http://www.cs.mu.oz.au/~mkwan/pendulum/pendulum.html>
Magnetic Pendulum—like the Wildwood pendulum in my office! Also, see
<http://www.cooper.edu/~wolf/chaos/wild.avi>
Rattleback: Garcia, A. and M. Hubbard (1988), Proc. Roy. Soc. London 418, 165-197
Sprung Pendulum: Rusbridge, M. G. (1979), Am. J. Phys. 48, 146-151.

16) ITLL Chaotic Pendulum
The chaotic pendulum in the lobby of the ITLL has recently been instrumented. This is a perfect opportunity to take some data, do "phase space embedding"(see (1)), compare the data to a pendulum model, etc. You might also use the "virtual" experiment at <http://physics.mercer.edu/pendulum>.

16) Nonlinear Circuits
Build a nonlinear oscillator to demonstrate the Period doubling route to chaos. See <http://amath.colorado.edu/faculty/jdm/faq-[3].html - Heading28>

17) Small Worlds
Recently it has been discovered that many networks have the "small world" property. For example, consider the world wide web. This is a network of connected pages. It apparently can be shown that it takes no more than 19 click to get from any page to any other page (http://physicsweb.org/article/world/14/7/09). Many more examples exist: See the book Watts, D. (1999), *Small Worlds* (Princeton Univ. Press, Princeton). Your task, should you accept the challenge, would be to study small worlds that have some relation to dynamics.

18) Economic cycles
Is there an economic cycle? Why do we have periods of prosperity and periods of recession? Check the following articles: Laaksonen, Matti. "Oscillations in some nonlinear economic relationships" (1996) Chaos, Solitons and Fractals 7 2235-2245. Szydowski, M., Krawiec, A. and Tobo, J. "Nonlinear oscillations in business cycle model with time lags" (2001) Chaos, Solitons and Fractals. 12: .505-517.

19) Models of economic growth
There are many models of economic growth, generally resulting from Dynamic Optimization, that can be written in terms of a system of differential equations. For instance, one can study the models of Solow and Ramsey. Check the following books: Shone, R, Economic Dynamics (1997) Cambridge University Press, Klein, M. Mathematical Methods for Economics (1998) Addison-Wesley or Chiang, A. Dynamic Optimization (1992) McGraw-Hill.