APPM 3310: Matrix Methods — Exam #1 — June 19, 2007

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes and calculators are not permitted. **Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.**

1. (20 points)

   (a) Give the definitions for the range and the kernel of an $m \times n$ matrix A.

   (b) Find a basis and the dimension for ker(A) and rng(A), for the following matrix:

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 4 \\ 1 & -2 & -1 \\ -1 & 1 & -1 \end{pmatrix}$$

2. (30 points)

   (a) Is the following matrix invertible?(Please, explain why or why not.)

$$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 3 & 2 \\ -1 & 2 & 0 \end{pmatrix}$$

   (b) Write A in (if necessary, permuted) LDV form.

   (c) Solve the system $Ax = b$, for $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

3. (15 points) Consider a square matrix A.

   (a) Show that $A + A^T$ is a symmetric square matrix (where A^T denotes the transpose of A).

   (b) Show that $A - A^T$ is a skew symmetric square matrix. (Recall that a matrix K is skew symmetric if $K^T = -K$.)

   (c) Show that A can be written in the form $A = S + K$, where S is symmetric and K is skew symmetric.

4. (40 points) Each of the following statements is either True or False? Please, justify your "True" answers with a proof and your "False" answers with a counterexample.

   (a) The vectors $\begin{pmatrix} 2 \\ 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 2 \\ -3 \end{pmatrix}$ are linearly independent in $\mathbb{R}^4$.

(b) The subset of $n \times n$ symmetric matrices is a subspace of the vector space of all square $n \times n$ matrices.

(c) The subset of $n \times n$ nilpotent matrices is a subspace of the vector space of all square $n \times n$ matrices. (A nilpotent matrix is a matrix A for which there exists a $k \in \mathbb{N}$ such that $A^k = 0$.)

(d) tr(AB)=tr(BA), for any $m \times n$ matrix A and any $n \times m$ matrix B.

5. (10 points) Consider the matrix:

$$C = \begin{pmatrix} a & a & a \\ a & b & b \\ a & b & c \end{pmatrix}$$

When (i.e., under what conditions for a, b and c) do the columns of C form a basis for $\mathbb{R}^3$? (Hint:A collection of n vectors forms a basis of $\mathbb{R}^n$ iff the matrix having the respective vectors as its columns has rank n.)