

APPM 3310: Matrix Methods — Exam #1 — February 23, 2007

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes and calculators are not permitted.

**Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.**

1. (40 points) Let  $A$  be an  $m \times n$  matrix.
  - (a) Give the definitions for the range of  $A$  and the kernel of  $A$ .
  - (b) State the Fundamental Theorem of Linear Algebra, part 1.
  - (c) Now find a basis and the dimension for each of the four fundamental subspaces of the matrix  $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 2 & 4 \end{pmatrix}$
  - (d) Let  $V$  be the span of  $v_1 = (1, 0, 1, -1)^T$ ,  $v_2 = (-1, 2, -2, 1)^T$ , and  $v_3 = (1, 4, -1, -1)^T$ . Does  $V = \ker(A)$ ? Explain.
2. (15 points) Give conditions on vector  $b$  for there to be one, none, and/or infinitely many solutions in the following matrix equation. Briefly explain your reasoning, but you don't need to find the solution(s).  $Bx = \begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$
3. (30 points) Let  $C = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$ 
  - (a) Find matrices  $L$ ,  $D$ , and  $V$  so that  $C = LDV$ , where  $L$  is lower triangular,  $D$  is diagonal, and  $V$  is upper triangular.
  - (b) Find conditions on  $a$ ,  $b$ , and  $c$  so that matrix  $C$  is invertible.
  - (c) Can you find an expression for the inverse for  $C$ ? (Hint: Do not do Gauss-Jordan on matrix  $C$ . Find  $L^{-1}$  and use that.)
4. (15 points) Give a complete explanation for each of the following.
  - (a) Let  $A$  be any  $n \times n$  matrix. Show that  $K = A - A^T$  is skew-symmetric. (Recall, a matrix  $K$  is skew-symmetric if  $K = -K^T$ .)
  - (b) If  $\ker(B) = \{\mathbf{0}\}$  and  $B$  is  $n \times n$ , then  $B$  is invertible.

Extra Credit (5 points): Assume  $A$  is an  $n \times n$  nilpotent matrix. Characterize the inverse of  $A$ . (A complete answer will include the definition of a nilpotent matrix and an explanation for your answer.)