Student Project, APPM 3310 — Summer 2007

Final Project due dates: June 18, July 3, and July 26.

In this class, we study some of the foundations of linear algebra and different types of applications. In this project, choose an application of matrix methods (of interest to you!) and prepare a paper on this topic. Some possible topics (in no particular order) include:

1. Graphs and incidence matrices from section 2.6.
2. Image compression. This is a huge area of active research, but it is fundamentally a problem in linear algebra. If you are interested in this, you will need to restrict to a specific subtopic.
3. There are many "named" matrix families. These include: Hadamard, Hankel, Hilbert, Magic, Pascal, Rosser, Toeplitz, and Vandermonde. You can investigate one of these families—applying many of the techniques that we are learning—and their application.
4. Springs and masses in section 6.1.
5. Electrical networks in section 6.2.
6. Structures in sections 6.3 and 9.5.
7. Additional topics in chapters 9 and 11.
8. Linear programming. (This is a huge area, you'll need to restrict to one small set of examples.)
9. Network models and spanning trees—such models are important for an understanding of the internet, of power distribution (and blackouts), etc.
10. Linear economic models.
11. Wavelets are based on matrix methods. You might investigate the Haar wavelet family. There are several "introductory" texts on wavelets, including *Discovering Wavelets* by Edward Aboufadel and Steven Schlicker.
12. The vector space model is fundamental to a basic understanding of search engines. *Understanding Search Engines* by Michael Berry and Murray Browne. is a place to start.
13. The fast Fourier transform (FFT) is a mainstay in computational linear algebra. One place to learn more is section 5.7.

These topics are not exhaustive. Come talk with me about your ideas. I'd like you to choose a topic that interests you! Here are a few titles of past very successful projects:

- *Application of linear algebra in structural analysis.* The paper examines the use of linear algebra in the analysis of simple truss structures. The deformation, or stiffness method, is outlined, and the practical case of a statistically indeterminate truss is given as an example.
- *The discrete cosine transform.* The DCT and the Inverse DCT form the basis of some of the most popular forms of media compression. The paper discusses DCT, with emphasis on the use of Matlab to implement a basic image compression scheme.
- *Analyzing adjacency matrices to determine winners in athletic tournaments.* In a tournament, competing team are represented, in graph theory, as vertices, and matches are represented as edges between winners and losers. By adapting the HITS method (that certain search engines use to rank the connectivity of websites) one can determine the "hub" scores for each team.

- *Computational costs of solving a circuit* Electrical networks provide a good application for matrices as a way to compute solutions to the generated systems of equations. The paper focuses on three methods: Gaussian Elimination, Cramer's Rule and using inverses.

- *It will warp time, space, and your mind: the mathematics of special relativity.* Deals with special relativity and the idea that simultaneity is not the same for all observers. Using mathematics, the paper quantifies some realistic situations, converting them into systems of linear equations.

The paper will proceed in three steps. The first part will be a 2-page project proposal (Due June 18). The project proposal will include an introduction to your topic, some background information, an outline of what you will cover in your paper, and a start on your list of references. The second part will be a rough draft (Due July 3) and the third part will be the final paper (Due July 26).

Your project will be written in a manner similar to a research or journal article. It should be typewritten, although lengthy derivations can be handwritten and included in an appendix. Your paper should have the following sections, as they pertain to your problem. (Your paper should not be a random collection of facts. You should identify at least one specific question/problem. The goal of your paper is to answer that question.)

Abstract: A short (1-2 paragraphs) summary of your problem and results.

Introduction: This section serves to introduce your topic and provide any background information. Include a survey of other people's work and/or a short discussion of the existing literature, as appropriate. Provide all necessary definitions and concepts for me (presumably ignorant of your particular application) to understand the question you are studying. Provide references as appropriate. (This section may be anywhere from 2-5 pages long.)

Model development/mathematical formulation: This section serves to develop your model. All symbols should be defined/identified. Any approximations you make in going from your physical model to the mathematical model should be noted. Lengthy derivations can be included here or placed in an appendix. Results (and proofs, where appropriate) can be included in this section or can stand alone in their own section.

Numerical Work (if appropriate) or Examples: This section describes your algorithms and/or numerical work and/or examples. A descriptive summary of your code and results can be included here. You can put any (well-documented!) code that you write in the appendix. Graphs and/or tables with data you used or computed should be in this section.

Discussion/Conclusions: What did you find? Is it what you had expected? How can you use your results? What do they tell you about your original question?

References All books, articles, and websites used in the preparation of your paper should be documented in this section. Note that if you do not properly document your references and attribute the results that you used to the proper sources, then your work is plagiarism.

Additional items to keep in mind:

- Talk with me. I want to hear your ideas!

- You can work alone or create small study groups.

- The project will represent up to 100 points toward the total course requirement of 450.

- Cover pages are not needed, just a title, date, your name at the top of the page. You can also print 2-sided if you wish.
- Give complete definitions of all terms and symbols used.
- Figures, graphs, and tables should have labels, captions and legends, as appropriate. You should discuss each figure in the text, as in "We see from Figure 3 that ..." If you do not discuss it in the text, it should not be included in the report!
- Reference facts that you state in your paper. Include all references at the end of the paper.
- In grading your project, I will give roughly equal weight to each of the following:
  - Writing style, clarity and completeness of explanations.
  - Problem choice, mathematical development and explanation of the problem.
  - Numerical development and/or examples, explanation and discussion. In general, code should be in an appendix and graphs and figures should be in the text where they are first discussed.