Department of Applied Mathematics
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION
January 2005

Notice: Do four of the following five problems. Place an X on the line opposite the number of the problem that you are **NOT** submitting for grading. Please do not write your name anywhere on this exam. You will be identified only by your student number, given below and on each page submitted for grading. Show all relevant work.

1. ____
2. ____
3. ____
4. ____
5. ____

Total ____

Student Number ____________________________

1. Suppose that X_1, X_2, and X_3 have a *trinomial* distribution with index n and probability parameters p_1, p_2, and p_3 where $\sum p_i = 1$. The log-likelihood function is

$$l(p_1, p_2, p_3) = \sum X_i \log p_j,$$

and the observed values of the X_j's are 32, 46, and 22.

   (a) Find the maximum likelihood estimates of the p_j's.

   (b) Find the maximum likelihood estimates of the p_j's, when the p_j's satisfy the hypothesis

$$p_1 = \theta^2, \qquad p_2 = 2\theta(1-\theta), \qquad p_3 = (1-\theta)^2.$$

2. Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed random variables with rate parameter λ. Let N be a geometric random variable with parameter p. Assume that the X's and N are independent. Let $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$. Find the following.

   (a) $\mathsf{E}\left[\frac{1}{N}\sum_{i=1}^{N} X_i\right]$

   (b) $P(X_{(N)} > a)$

   (c) $\mathsf{E}\left[X_{(N)}\right]$

3. Let X_1 and X_2 be a random sample from the geometric distribution with probability density function given by

$$f(x) = P(X = x) = p(1-p)^x, \qquad x = 0, 1, 2, \ldots$$

where $0 < p < 1$.

   (a) Find the UMVUE for $(1+p)^2$.

   (b) Find the UMVUE for $(1+p)^2$/p.

4. Let X, Y, and U be independent random variables with $X \sim Poisson(\lambda)$, $Y \sim Poisson(\mu)$ and $U \sim Uniform(0,1)$. Let

$$X = \begin{cases} X & , \text{ if } U > a \\ Y & , \text{ if } U \leq a. \end{cases}$$

Find

$$P(X + Y = n | V = k).$$

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from the $N(\mu, \sigma^2)$ distribution where $\sigma^2 > 0$ is known.

   (a) Find the generalized likelihood ratio test (GLRT) of size α for testing $H_0 : \mu = \mu_0$ versus $H_a : \mu \neq \mu_0$.

   (b) Find the exact (not asymptotic) distribution of $-2\lambda(\vec{X})$ where $\lambda(\vec{X})$ is the (generalized) likelihood ratio (GLR) you used in part (a).

6. Assume that a beginning student going to a certain medical school has, each year, a probability 0.1 of flunking out, a probability 0.2 of having to repeat the year, and a probability 0.7 of moving on to the next year. (In the fourth year, moving on means graduating.)

   (a) How long should this student expect to be in medical school.

   (b) Find the probability that the student will graduate.