Mathematical Harmonies
Worksheet for Graphing Calculators

Example 1. A simple note can be displayed as a graph of pressure verses time using:

$$P = A\sin(2\pi ft)$$

where: *P* is pressure
t is time
A is amplitude (height of the wave) or volume
f is the frequency or pitch.

- On your calculator, graph y1=6sin(2π0.3x) and zoom standard.
- To make a louder note, increase the amplitude: change the 6 to a 9.
- To make a higher pitch, increase the frequency: change the 0.3 to a 0.5

Audible frequencies range from 20 to 20,000 Hz, but 0.3 is used here for a good display.

Example 2. Create the "signature" pressure waves of a flute by adding the fundamental frequency and the second harmonic.

- Graph y1=4sin(2π0.2x) and y2=3sin(2π0.4x) Here y1 is the fundamental and y2 is the second harmonic. Notice that the frequency of y2 is twice that of y1, and y2 is little quieter than y1.
- To see the "signature", add the two waves together: y1=4sin(0.4 πx)+3sin(0.8πx)

This is what a flute's sound would look like on an oscilloscope (pressure measuring device)

Example 3. When two notes are played with slightly different pitches, beats can be heard. This is caused by the superposition (adding together) of the waves.

- Graph y1=4sin(2π0.6x) and y2=4sin(2π0.7x). Notice how the waves interweave.
- Now add them: y1=4sin(2π0.6x)+4sin(2π0.7x). Note the super waves, which are the beats.
- Change the 0.7 to 0.8. As the frequencies move apart, the waves get faster.
- Now change the 0.8 to 0.9. As the frequencies move farther apart, the super waves become indistinct.

Beats can be heard when the pitches of two notes are within a half-step.