**Mathematical Harmonies**

**Worksheet for Graphing Calculators**

**Example 1.** A simple note can be displayed as a graph of pressure verses time using:

\[ P = A \sin(2\pi ft) \]

where:
- \( P \) is pressure
- \( t \) is time
- \( A \) is amplitude (height of the wave) or volume
- \( f \) is the frequency or pitch.

- On your calculator, graph \( y_1 = 6\sin(2\pi 0.3x) \) and zoom standard.
- To make a louder note, increase the amplitude: change the 6 to a 9.
- To make a higher pitch, increase the frequency: change the 0.3 to a 0.5

Audible frequencies range from 20 to 20,000 Hz, but 0.3 is used here for a good display.

**Example 2.** Create the “signature” pressure waves of a flute by adding the fundamental frequency and the second harmonic.

- Graph \( y_1 = 4\sin(2\pi 0.2x) \) and \( y_2 = 3\sin(2\pi 0.4x) \) Here \( y_1 \) is the fundamental and \( y_2 \) is the second harmonic. Notice that the frequency of \( y_2 \) is twice that of \( y_1 \), and \( y_2 \) is little quieter than \( y_1 \).
- To see the signature, add the two waves together: \( y_1 = 4\sin(0.4 \pi x) + 3\sin(0.8 \pi x) \)

This is what a flute’s sound would look like on an oscilloscope (pressure measuring device).

**Example 3.** When two notes are played with slightly different pitches, beats can be heard. This is caused by the superposition (adding together) of the waves.

- Graph \( y_1 = 4\sin(2\pi 0.6x) \) and \( y_2 = 4\sin(2\pi 0.7x) \). Notice how the waves interweave.
- Now add them: \( y_1 = 4\sin(2\pi 0.6x) + 4\sin(2\pi 0.7x) \). Note the super waves, which are the beats.
- Change the 0.7 to 0.8. As the frequencies move apart, the waves get faster.
- Now change the 0.8 to 0.9. As the frequencies move farther apart, the super waves become indistinct.

Beats can be heard when the pitches of two notes are within a half-step.