1. Consider the Markov chain on $S = \{0, 1, 2\}$ running according to the transition probability matrix

$$
P = \begin{bmatrix}
0 & 1 & 2 \\
0 & \frac{1}{3} & \frac{2}{3} \\
\frac{3}{4} & \frac{1}{4} & 0 \\
\end{bmatrix}
$$

(a) Starting in state 0, what is the mean time that the process spends in state 1 prior to first hitting state 2?

(b) Starting in state 0, what is the mean time that the process spends in state 1 prior to returning to state 0?

2. A rat with a bad cold (so he can’t smell!) is put into compartment 4 of the maze shown in Figure ???. At each time step, the rat moves to another compartment. (He never stays where he is.) He chooses a departure door from each compartment at random. What is the probability that he finds the food before he gets shocked.

3. (purple Durrett 7.2) The lifetime of a radio is exponentially distributed with mean 5 years. If Ted buys a 7 year-old radio, what is the probability it will be working 3 years later?

4. (Most of purple Durrett 7.6) Consider a bank with two tellers. Three people, Alice, Betty, and Carol enter the bank at almost the same time and in that order. Alice and Betty go directly into service while Carol waits for the first available teller. Suppose that the service times (in minutes) for each customer are exponentially distributed with rate $1/4$.

(a) What is the expected total amount of time for Carol to complete her business transaction? (Include waiting and service time.)
(b) What is the expected total time until the last of the three customers leaves?
(c) What is the probability that Carol is the last one to leave?

5. (purple Durrett 7.32) Let $T_1, T_2, \ldots, \text{iid} \sim \exp(rate = \lambda)$ and let $N$ be an independent random variable with $P(N = n) = p \cdot (1 - p)^{n-1} I_{\{1,2,\ldots\}}(n)$. (That is, $N \sim \text{geom}_1(p)$.) What is the distribution of $T = T_1 + T_2 + \cdots + T_N$?

6. (Durrett 7.22) Suppose that the number of calls per hour to an answering service follows a Poisson process with rate 4.

(a) What is the probability that fewer than 2 calls come in during the first hour?
(b) Suppose that 6 calls arrive in the first hour. What is the probability that less than 2 calls come in during the second hour?
(c) Given that exactly 6 calls arrive during the first two hours, what is the conditional probability that exactly 2 arrived during the first hour and exactly 4 arrived in the second hour?
(d) Suppose that the operator gets to take a break after she has answered 10 calls. How long are her average work periods?