Problem #1 (6 points): Let

\[
A = \begin{pmatrix}
-1 & 3 & 0 & 3 \\
0 & -3 & 2 & 0 \\
-2 & 0 & 7 & 3 \\
\end{pmatrix}
\quad \text{and} \quad
b = \begin{pmatrix}
2 \\
2 \\
\end{pmatrix}
\]

(a) Find the LU decomposition of A.
(b) Find all the solutions of \(Ax = 0\).
(c) Using your LU decomposition, find all the solutions of \(Ax = b\).

Problem #2 (6 points): Let \(A\) be defined as in problem #1.

(a) Find bases for ker \(A\), coker \(A\), rng \(A\), and corng \(A\).
(b) Find rank(\(A\)), rank(\(A^T\)), dim(ker \(A\)), dim(coker \(A\)), dim(rng \(A\)), and dim(corng \(A\)).

Problem #3 (6 points): Let \(S = \{2x^2 - 1, 4x^2 - 1, \frac{3}{2}x^2 - \frac{1}{2}, \frac{1}{2}x^2 - 2x + 1\}\).

(a) Does \(S\) span \(P_2 = \{p(x) = a_0 + a_1x + a_2x^2 : a_0, a_1, a_2 \in \mathbb{R}\}\)?
(b) Is the set \(S\) linearly independent or linearly dependent? If \(S\) is linearly dependent, give a subset of \(S\) that both spans the same space and is linearly independent.

Problem #4 (6 points): Answer the following with “True” or “False” – don’t just put “T” or “F”. If false, give a counterexample.

(a) If \(A\) has \(m\) rows and \(n\) columns, then the nullspace of \(A\) is a subspace of \(\mathbb{R}^m\).
(b) Let \(A\) be a square matrix. If \(Ax_1 = Ax_2\) and \(x_1 \neq x_2\), then \(A\) is not invertible.
(c) Let \(A\) be a square matrix. If \(x \in \ker(A^2)\), then \(x \in \ker(A)\).
(d) Let \(A\) be a square matrix. If \(\ker(A) = 0\) then \(\det A \neq 0\).
(e) If \(AB = I\), then \(A\) and \(B\) are square matrices.
(f) \(V = \text{span}\{1, x, x^4\}\) is a subspace of the vector space of all polynomials of degree less than or equal to 4?