On the front of your bluebook, write your name and make a grading table. You’re allowed one sheet (letter-sized, front and back) of notes. You are not allowed to use textbooks, class notes, or a graphing calculator, but you can use a scientific calculator.

**Problem #1 (8 points):**
(a) Find the line $y(x) = a + bx$ that gives the least squares approximation to \{(0, 1), (1, 1), (2, 2)\}.
(b) Find a matrix representation for a rotation in $\mathbb{R}^3$ through angle $\theta$ about the $x$-axis.
(c) Find the $\text{rng}(A^T)$ and $\ker(A)$, where $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$.
(d) For $A$ in part (c), show that $\text{rng} A^T = (\ker A)^\perp$.

**Problem #2 (6 points):**
(a) Give a definition for a positive-definite matrix.
(b) For what values of $\alpha$ is \[ K = \begin{pmatrix} 2 & 1 & -1 \\ 1 & \alpha & 0 \\ -1 & 0 & 1 \end{pmatrix} \] positive definite?
(c) Find the Cholesky decomposition of $K$ for $\alpha = 2$.

**Problem #3 (6 points):** Find the QR factorization of \[ A = \begin{pmatrix} -2 & 4 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & 5 \end{pmatrix} \] and verify that $Q$ is orthogonal.

**Problem #4 (4 points):** Consider the inner product \[ \langle f, g \rangle = \int_{-\infty}^{\infty} f(x) g(x) e^{-x^2} \, dx. \]
Set $H_0(x) = 1$. Use the Gram-Schmidt process to find orthogonal polynomials of degree one, two, and three with respect to this inner product.

Note that it is sufficient to construct an orthogonal sequence – you do not need to normalize. You may use
\[ \int_{-\infty}^{\infty} x^j e^{-x^2} \, dx = \begin{cases} 0, & j \text{ odd} \\ \Gamma \left( \frac{1+j}{2} \right), & j \text{ even} \end{cases} \]
and that $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(3/2) = \sqrt{\pi}/2$, and $\Gamma(5/2) = 3\sqrt{\pi}/4$. 

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