If we're so different, why do we keep overlapping? When 1 plus 1 doesn't make 2

Rory Wolfe, James Hanley

In the last decade, guidelines for the presentation of statistical results in medical journals have emphasized confidence intervals (CIs) as an adjunct to, or even a replacement for, statistical tests and *p* values. Because of the intimate links between the 2 concepts, authors now use statements like "the 95% CI overlaps 0" where they would formerly have stated "the difference is not statistically significant at the 5% level." Although this interchangeability is technically correct in 1-sample situations, it does not carry over fully to comparisons involving 2 samples. A frequently encountered misconception is that if 2 independent 95% CIs overlap each other, as they do in Fig. 1, then a statistical test of the difference will not be statistically significant at the 5% level.

Why is this not necessarily so? Consider the means in 2 independent groups, $mean_A$ and $mean_B$, with for simplicity $mean_A$ being the smaller of the 2. The 95% CI for the mean in group A is approximately given by $mean_A$ plus or minus twice the standard error of the mean for that group, SE_A, and correspondingly for group B. A mathematical check for whether these CIs overlap is given by adding the distance $2SE_A$ (from $mean_A$ to the upper bound of the CI) to $2SE_B$ and comparing this sum with the distance between the 2 means, that is, $mean_B$ minus $mean_A$ (Fig. 2). The CIs overlap when

$$mean_B - mean_A < 2SE_A + 2SE_B \quad [1]$$

But overlapping confidence intervals do not demonstrate that group means are not statistically significantly different from each other. In a 2-sample *t*-test to compare 2 means, significance is attained at the 0.05 level if the *t* statistic exceeds the critical value of about 2, which occurs when the difference between the means exceeds twice its standard error, namely, if

$$mean_B - mean_A > 2\sqrt{(SE_A^2 + SE_B^2)} \quad [2]$$

This standard error reflects the fact that the standard error of a difference involves summing the standard error of each estimate, but doing so by "adding in quadrature," for example,

$$1 \text{ “+” } 1 = \sqrt{(1^2 + 1^2)} = 1.414 \quad [3]$$

Thus, to evaluate the overlap of 2 95% CIs and to determine whether at the same time the difference between the

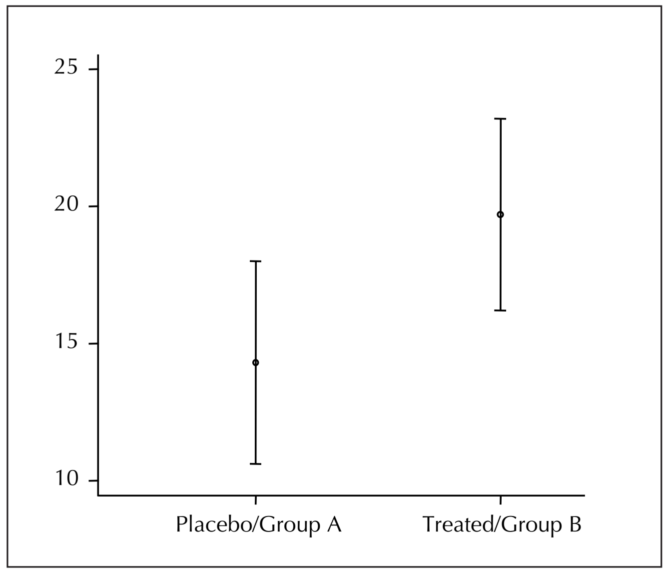

Fig. 1: Group means with confidence intervals that overlap.

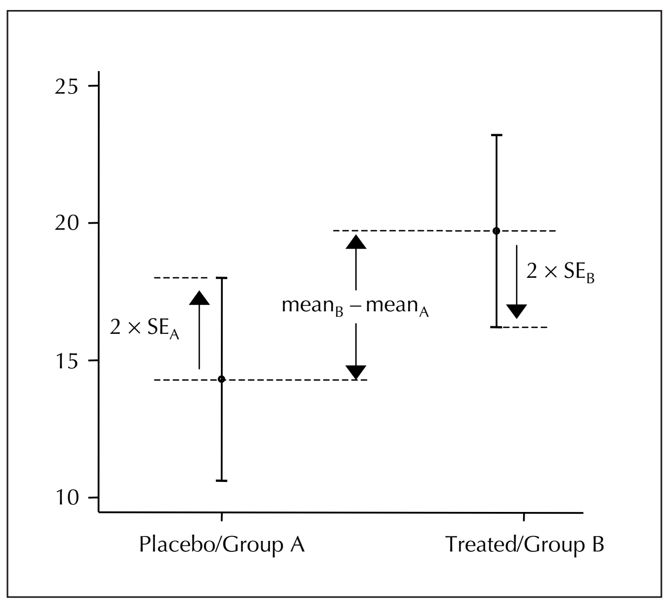

Fig. 2: Confidence intervals and comparison of 2 group means (hypothetical clinical trial data: $SE_A = SE_B = 1.8$, means differ by 3 SE; assuming $n > 30$ and independent samples, the 2-sided *p* value for testing the difference in means is approximately 0.036). SE = standard error of the mean.

means is significant at the 0.05 level, the following rough rule can be used:

$$[4] \quad 2\sqrt{(SE_A^2 + SE_B^2)} < mean_B - mean_A < 2SE_A + 2SE_B$$

If SE_A and SE_B are equal, the condition is as follows:

$$[5] \quad 2.83\ SE < mean_B - mean_A < 4\ SE$$

When one SE is 25% larger than the other, the boundaries are 3.2 and 4.5 times the smaller SE. As the lower boundary remains close to 3, Moses[1] was prompted to display group means with error bars that were 1.5 SE around the mean in order to have a "by eye" test of significance between the 2 group means while presenting the information in the 2 groups separately.

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Dr. Wolfe is with the Department of Epidemiology and Preventive Medicine, Monash University, Melbourne, Victoria, Australia. Dr. Hanley is with the Department of Epidemiology and Biostatistics, McGill University, Montreal, Que.

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Correspondence to: *Dr. Rory Wolfe, Department of Epidemiology and Preventive Medicine, Central and Eastern Clinical School, Monash University and the Alfred Hospital, Commercial Rd., Prahran, Victoria 3183, Australia; fax 61 3 9903 0556; rory.wolfe@med.monash.edu.au*

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